# UZLUKSIZ FUNKSIYANING MODULI

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## ANNOTATSIYA

Maqolada $[a,b]$ kesmada chegaralangan $f(x)$ funksiyaning uzluksizlik moduli va uning asosiy xossalari tahlil qilinadi.

**Kalit so'zlar:** funksiya, uzluksiz funksiya, matematik induksiya, Gyolder fazosi.

## MODULE OF CONTINUOUS FUNCTION

## ABSTRACT

The article analyzes the continuity module of the function f(x) bounded on the section [a,b] and its main properties.

**Keywords:** function, continuous function, mathematical induction, Golder space.

$[a,b]$ da chegaralangan $f(x)$ funksiya berilgan bo'lsin. Ushbu

$$\sup_{\substack{|x_1-x_2|<\delta \\ a\le x_1,x_2\le b \\ 0<\delta\le b-a}} |f(x_2)-f(x_2)| \stackrel{\text{def}}{=} \omega(f,\delta)$$

funksiyaga $f(x)$ funksiyaning modul uzluksizligi deyiladi.

Takidlaymizki, $\lim_{\delta\to 0}\omega(f,\delta)=0$ shart $f(x)$ funksiyaning uzluksiz bo'lishi uchun zaruriy va yetarli shart bo'ladi.

$[a,b]$ da uzluksiz bo'lgan funksiyalar sinfini $C_{[a,b]}$ deb belgilaymiz. Biz bundan keyin $f(t)\in C_{[a,b]}$ deb qaraymiz.

Uzluksiz funksiyaning modul uzluksizligi ta'rifidan uning quyidagi xossalari kelib chiqadi:

**$1^0$.** $\omega(f,0)=0$

**$2^0$.** $\omega(f,\delta)$ funksiya $\delta$ bo'yicha kamayuvchi.

**$3^0$.** $\omega(f,\delta)$ yarim additiv, ya'ni

$$\omega(f,\delta_1+\delta_2)\le\omega(f,\delta_1)+\omega(f,\delta_2)$$

**$4^0$.** $\omega(f,\delta)$, $\delta$ bo'yicha $[a,b]$ da uzluksiz funksiya bo'ladi.

**1-ta'rif.** Agar $\omega(\delta)$ $(0 < \delta \leq l_0 = b - a)$ funksiya $1^0 - 4^0$shartlarni qanoatlantirsa, u holda uzluksiz funksiyaning modul uzluksizligi deyiladi.

**1-lemma.** $\omega(\delta)$ $(a, l_0]$ da $\uparrow$, $\varphi(\delta) \geq 0$, yarim additiv bo'lsa, u holda $\forall t_1, t_2 \in (a, l_0]$ uchun $|\varphi(t_1) - \varphi(t_2)| \leq \varphi C|t_1 - t_2|$ o'rinli.

$\omega(\delta)$-modul uzluksiz bo'lsin. U holda $\omega(f, \delta) = \omega(\delta)$ bo'ladi.

Haqiqatdan ham

$$\omega(\omega, \delta) = \sup_{|\delta_1 - \delta_2| < \delta} |\omega(\delta_1) - \omega(\delta_2)| \leq \sup_{|\delta_1 - \delta_2| < \delta} \omega(|\delta_1 - \delta_2|) \leq \omega(\delta)$$

ikkinchi tomondan $\omega(\omega, \delta) \geq \omega(\delta) - \omega(0) = \omega(\delta)$,

ya'ni $\omega(\omega, \delta) = \omega(\delta)$.

**2-lemma.** Agar $[0, b - a]$ da kamaymovchi, uzluksiz bo'lgan $\omega(\delta)$ funksiya: $(0) = 0$ , $\frac{\varphi(\delta)}{\delta}$ o'smovchi bo'lsa, u holda $\omega(\delta)$-modul uzluksiz bo'ldi.

**Isbot.** $\varphi(\delta)$ funksiya uchun $1^0, 2^0$ va $4^0$ lar lemmaning shartiga ko'ra bajariladi. $3^0$ munosabat quyidagi

$$\varphi(\delta_1 + \delta_2) = \delta_1 \cdot \frac{\varphi(\delta_1 + \delta_2)}{\delta_1 + \delta_2} + \delta_2 \cdot \frac{\varphi(\delta_1 + \delta_2)}{\delta_1 + \delta_2} \leq$$

$$\leq \delta_1 \cdot \frac{\varphi(\delta_1)}{\delta_1} + \delta_2 \cdot \frac{\varphi(\delta_2)}{\delta_2} = \varphi(\delta_1) + \varphi(\delta_2).$$

Takidlaymizki, $\frac{\varphi(\delta)}{\delta}$ ning o'smovchi bo'lishlik sharti modul uzluksiz uchun yetarli shart bo'lib hisoblanadi.

Modul uzluksizlikning navbatdagi xossasi:

**$5^0$.** Agar $\omega(\delta)$-modul uzluksiz bo'lsa, u holda $\forall \lambda \in R$ $(\lambda > 0)$ uchun

$$\omega(\lambda\delta) = (\lambda + 1)\omega(\delta) \qquad (1)$$

tengsizlik o'rinli bo'ladi.

**Isbot.** a) $\forall n \in N$ bo'lsin. Bu xossaning isboti $3^0$ xossadan bevosita kelib chiqadi, ya'ni

$\omega(\delta_1 + \delta_2) \leq \omega(\delta_1) + \omega(\delta_2) \Rightarrow \delta_1 = \delta_2 = \delta$

deb olsak,

$\omega(2\delta) \leq 2\, \omega(\delta)$

Matematik induksiya usuli yordamida $\omega(n\delta) \leq n\, \omega(\delta)$ tengsizlikning o'rinli ekanligini ko'rsatish mumkin.

b) $\forall \lambda \in R$ bo'lib, $(\lambda > 0)$ $n < \lambda < n + 1$ bo'lsin. Ma'lumki, $\omega(\delta)$-monoton o'suvchi funksiya. U holda $\omega(\lambda\delta) \leq \omega(n + 1)\delta \leq (\lambda + 1)\omega(\delta)$.

**$6^0$.** $\delta_1 \le \delta_2$ bo'lsin. U holda

$$\omega(\delta_2) = \omega\left(\delta_1 \cdot \frac{\delta_2}{\delta_1}\right) \le \left(\frac{\delta_2}{\delta_1} + 1\right)\omega(\delta_1) = \frac{\delta_2}{\delta_1}\left(1 + \frac{\delta_1}{\delta_2}\right)\omega(\delta_1) \le 2\frac{\delta_2}{\delta_1}\omega(\delta_1).$$

Keyingi tengsizlikdan $\forall\delta_1 \le \delta_2$ bo'lganda

$$\frac{\omega(\delta_2)}{\delta_2} \le 2\frac{\omega(\delta_1)}{\delta_1} \qquad (*)$$

bo'ladi. Bu xossadan $\frac{\omega(\delta)}{\delta}$ –deyarli kamaymovchi funksiya ekanligi kelib chiqadi.

$\varphi(\delta)$ $(0, l_0]$ da aniqlangan musbat funksiya bo'lsin.

**2-ta'rif.** Agar $\exists A > 0$ $(A_1 > 0)$ topilib, $\forall 0 < \delta_1 < \delta_2$ lar uchun $\varphi(\delta_1) \le A\varphi(\delta_2)$ $(\varphi(\delta_1) \ge A\varphi(\delta_2))$ bo'lsa, u holda $\varphi(\delta)$ funksiya $(0, l_0]$ da deyarli o'suvchi (deyarli kamayuvchi) deyiladi.

**3-lemma.** Agar $\omega(\delta)$-modul uzluksiz bo'lsa, u holda

$$\frac{1}{2}\varphi(\delta) \le \frac{1}{\delta}\int_0^\delta \omega(t)dt \le \varphi(\delta) \qquad (2)$$

tengsizlik o'rinli bo'ladi.

**Isbot.** $\omega(\delta)$-modul uzluksiz bo'lsin.$\Longrightarrow \frac{1}{\delta}\int_0^\delta \omega(t)dt \le \frac{1}{\delta}\int_0^\delta \omega(\delta)dt \le \omega(\delta)$.

$$\omega(\delta) - \frac{1}{\delta}\int_0^\delta \omega(t)dt \le \frac{1}{\delta}\int_0^\delta (\omega(\delta) - \omega(t))dt \le \frac{1}{\delta}\int_0^\delta \omega(\tau)d\tau, \quad [\delta - t = \tau]$$

$$\frac{1}{2}\varphi(\delta) \le \frac{1}{\delta}\int_0^\delta \omega(\tau)d\tau \ .$$

**4-lemma.** Agar $\omega(\delta)$-modul uzluksiz bo'lsa, u holda

$$\frac{1}{x+y}\int_0^{x+y} \omega(t)dt \le \frac{1}{x}\int_0^x \omega(t)dt + \frac{1}{y}\int_0^y \omega(t)dt \qquad (3)$$

(bunda $0 \le x, y, x + y \le l_0$) tengsizlik o'rinli bo'ladi.

**Isbot.** $(3)$ da $y = \alpha x$ deb olish bilan topamiz:

$$\frac{\int_0^{(1+\alpha)x} \omega(t)dt}{1+\alpha} \le \frac{\int_0^{\alpha x} \omega(t)dt}{\alpha} + \int_0^x \omega(t)dt \qquad (4)$$

(4) ning to'g'riligi ushbu

$$\psi(x) = \int_0^x \omega(t)dt + \frac{1}{\alpha}\int_0^{\alpha x} \omega(t)dt - \frac{1}{1+\alpha}\int_0^{(1+\alpha)x} \omega(t)dt$$

funksiyaning $x = 0$ da $\psi(0) = 0$ bo'lishi va uning kamaymovchi ekanligidan kelib chiqadi.

**1-teorema.** Agar $\omega(\delta)$-modul uzluksiz bo'lsa, u holda

$$\omega^*(\delta) = \frac{1}{\delta}\int_0^\delta \omega(t)dt$$

funksiya ham modul uzluksiz bo'ladi.

**Isbot.** Agar (2) tengsizlikni e'tiborga olsak, u holda

$$\lim_{\delta\to 0}\omega^*(\delta)=0$$

ekanligiga ishonch hosil qilish qiyin emas. 2-lemmaga asosan $(\omega^*(\delta))'\geq 0$ , ya'ni $\omega^*(\delta)$-kamaymovchi funksiya. Ravshnki $\omega^*(\delta)$ –uzluksiz. $\omega^*(\delta)$ ning yarm additivligi 3-lemmadan kelib chiqadi. Shu bilan teorema isbot bo'ldi.

$\varphi(\delta)$ va $\psi(\delta)$ funksiyalar $(0,l_0]$ da aniqlangan noldan farqli, musbat uzluksiz bo'lsin.

**3-ta'rif.** Agar $\exists A_1,A_2>0$ sonlar mavjud bo'lib, $\forall\delta_1,\delta_2\in(0,l_0]$ lar uchun

$$A_1\psi(\delta)\leq\varphi(\delta)\leq A_2\psi(\delta)$$

tengsizlik bajarilsa, u holda $\varphi(\delta)$ va $\psi(\delta)$ funksiyalar $(0,l_0]$ da ekvivalent $(\varphi\sim\psi)$ deyiladi.

**4-ta'rif.** Agar $\exists A>0$ soni mavjud bo'lib, $\forall 0<\delta_1<\delta_2<l_0$ lar uchun

$$\varphi(\delta_1)\leq A\varphi(\delta_2)$$

tengsizlik o'rinli bo'lsa, u holda $\varphi(\delta)$ funksiya $(0,l_0]$ da deyarli o'suvchi deyiladi.

**5-ta'rif.** Agar $\exists A_1>0$ son mavjud bo'lib, $\forall 0<\delta_1<\delta_2<l_0$ lar uchun

$$\varphi(\delta_1)\geq A_1\varphi(\delta_2)$$

tengsizlik bajarilsa, u holda $\varphi(\delta)$ funksiya $(0,l_0]$ da deyarli kamayuvchi deyiladi.

Takidlaymizki, agar $M\geq\varphi(\delta)\geq\alpha>0\quad(0\leq\delta\leq 1)$ bo'lsa, u holda u deyarli o'suvchi bo'ladi. Bu holda $=\frac{M}{\alpha}$.

Agar $\varphi(\delta)$-modul uzluksiz bo'lsa, u holda (*) tengsizlikdan $\frac{\varphi(\delta)}{\delta}$ ning deyarli kamayuvchi funksiya bo'lishligi kelib chiqadi.

Ravshanki, agar $\varphi(\delta)\sim\psi(\delta)$ bo'lib, $\varphi(\delta)$ ning deyrli o'suvchi (deyarli kamayuvchi) ligidan $\psi(\delta)$ ning deyarli o'suvchi (deyarli kamayuvchi) ligi kelib chiqadi.

**5-lemma.** $\varphi(\delta)$ funksiya deyarli o'suvchi (deyarli kamayuvchi) bo'lishligi uchun kamaymovchi (o'smovchi) funksiyaga ekvivalent bo'lishi zarur va yetarlidir.

**Isbot.** Yetarliligi. $\varphi(\delta)$ funksiya biror kamaymovchi $\psi(\delta)$ funksiyaga ekvivalent, ya'ni $\varphi(\delta)\sim\psi(\delta)$ bo'lsin. 2-ta'rifga ko'ra $\exists A_1,A_2>0$ sonlari mavjud bo'lib, $\forall 0<\delta\leq l_0$ lar uchun

$A_1\psi(\delta)\leq\varphi(\delta)\leq A_2\psi(\delta)$ tengsizlik bajariladi. $\delta_2<\delta_1$ bo'lsin.

U holda yuqoridagi tengsizlikdan

$\varphi(\delta_2) \leq A_2\psi(\delta_2) \leq A_2\psi(\delta_1) \leq A_2\frac{\varphi(\delta_1)}{A_1} = \frac{A_2}{A_1}\varphi(\delta_1)$ bo'ladi. Demak, $\varphi(\delta)$ funksiya deyarli o'suvchi.

Zarurligi. $\varphi(\delta)$ deyarli o'suvchi funksiya bo'lsin, ya'ni $\forall 0 < \delta_1 < \delta_2 \in (0, l_0]$ lar uchun

$$\varphi(\delta_1) \leq A\varphi(\delta_2) \qquad (5)$$

ushbu

$$\psi(\delta) = \sup_{0<\eta<\delta}\varphi(\eta) \qquad (6)$$

funksiyani tuzamiz.

Ravshanki, $\psi(\delta)$ funksiya kamaymovchi. Endi kamaymoovchi $\psi(\delta)$ funksiyaning $\varphi(\delta)$ funksiyaga ekvivalentligini ko'rstamiz: $\psi(\delta)$ funksiyaning tuzulishidan $\varphi(\delta) \leq \psi(\delta)$ . (5) dan $\forall\eta\ (0 < \eta < \delta)$ uchun

$\psi(\eta) \leq A\varphi(\delta) \Rightarrow \psi(\delta) = \sup_{0<\eta<\delta}\varphi(\eta) \leq A\,\varphi(\delta), \psi(\delta) \leq A\varphi(\delta)$.

(6) ni e'tiborga olib topamiz: $\frac{1}{A}\psi(\delta) \leq \varphi(\delta) \leq \psi(\delta)$ tengsizlik bajariladi. Keyingi tengsizlikdan $\varphi(\delta) \sim \psi(\delta)$ ekanligi kelib chiqadi.

Xuddi shunday yo'l bilan lemmaning ikkinchi qismi ham isbot qilindi, ya'ni : $\varphi(\delta)$ funksiya deyarli kamayuvchi bo'lishligi uchun uning biror o'smovchi funksiyaga ekvivalent bo'lishi zarur va yetarlidir.

Ushbu maqola umumlashgan Gyolder fazosida funksiyaning uzluksizlik moduli va uning asosiy xossalari o'rganishga bag'ishlangan.

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