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Creation of baby-universe

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We introduce a metric in a 4-dimensional spacetime which describes creation of baby-universe from initially flat spacetime. Such metric may be used for construction of a transient intra-universe wormhole.

Keywords: *baby-universe, wormhole.*

1 Introduction

The problem of creation and existence of wormholes has attracted the attention of researchers for many years. A wormhole is a compact region of spacetime with nontrivial topology. According to Visser [1] there are permanent (or quasipermanent) and transient wormholes. The (quasi-)permanent intra-universe wormhole is a compact region which topologically equivalent to $R \times \Sigma$, where Σ is a spacelike hypersurface of nontrivial topology. Such wormholes are essentially three-dimensional objects. The transient wormhole is created and destructed without having $R \times \Sigma$ topological structure. These wormholes are intrinsically four-dimensional objects. In this case if we consider spacelike hypersurface $x^0 = \text{const}$ then wormhole is absent. Such hypersurface is either simply connected or consist of two disconnected regions. Creation of transient intra-universe wormhole may be thought of as the creation of baby-universe, it's separation from parent-universe at some point of spacetime and pasting both universe at the another point of spacetime.At the interval from separation to pasting, both universes are moving in their own time trajectories. Such construction was considered in [2] and the energy of transient wormhole creation was estimated (see also [3]). However above mentioned transient wormhole metrics are not obtained.

The aim of the present paper is to construct the metric tensor of spacetime such that there is a creation of baby-universe.

2 Set of Ω-like curves

In Ref. [4] a three-parametric set of Ω-like curves $\Omega(x, u; \lambda, \mu, \nu) = 0$ was received. It describes as the sphere S^1 separates from initially flat one-dimensional space $\mathbb{R}$. In other words, this set can be thought as one-dimensional model of baby-universe creation. Let us rewrite the set of Ω-like curves to one parametric form. It is a simple performed procedure, if we assume that λ, μ and ν are sufficiently smooth functions of t. We accept that these dependencies have forms presented in the table 1. Here λ_0, μ_1 and k are positive constants such that $\mu_1 > \lambda_0$ and $k > 1$. Moreover, first and second derivatives of functions $\mu(t)$, $\nu(t)$ and $\lambda(t)$ are equal to zero at the points t_1, t_2, t_3 and $t = 0$. The functions $\tilde{\mu}(t)$ and $\tilde{\nu}(t)$ increase, and $\tilde{\lambda}(t)$ is a decreasing function. Herewith $\tilde{\mu}(0) = 0$, $\tilde{\mu}(t_1) = \mu_1$, $\tilde{\nu}(t_1) = \lambda_0^2/\sqrt{\lambda_0^2 + \mu_1^2}$, $\tilde{\nu}(t_2) = \lambda_0^2/\sqrt{\mu_1^2 - \lambda_0^2}$, $\tilde{\lambda}(t_2) = \lambda_0$, $\tilde{\lambda}(t_3) = 0$.

So we receive that above mentioned set of curves is determined by equation $\Omega(x, u; t) = 0$, where

$$\Omega = \begin{cases} G(x, u; t), & \text{if } 0 \le u \le \nu,\ x \ge 0, \\ S^1(x, u; t), & \text{if } u > \nu, \\ G(-x, u; t), & \text{if } 0 \le u \le \nu,\ x < 0, \end{cases} \tag{1}$$

$$S^1(x, u; t) =$$

$$= x^2 + \left(u - \frac{(\mu^2 + \lambda^2)^2 \nu^4 - \lambda^8}{4\mu^2 \lambda^2 \nu^3} \right)^2 - R^2$$

and $G(x, u; t) = (\lambda^2 - \mu^2)u^2 + 2\lambda\mu x u - \lambda^4$. Here radius $R(t)$ is defined by

$$R^2 = \frac{(\lambda^4 + (\mu^2 - \lambda^2)\nu^2)^2}{4\mu^2\lambda^2\nu^2} + \frac{((\mu^2 - \lambda^2)^2\nu^4 - \lambda^8)^2}{16\mu^4\lambda^4\nu^6}. \tag{2}$$

The sphere $S^1(x, u; t) = 0$ tangents to the hyperbola $G(x, u; t) = 0$ in a point $M_\nu = (x_{\lambda\mu}(\nu), \nu)$, where $x_{\lambda\mu}(\nu) = \lambda^4 - (\lambda^2 - \mu^2)\nu^2/2\mu\lambda\nu$. This point may be also described by condition $u = \nu$. Functions $\lambda(t)$, $\mu(t)$, $\nu(t)$ evolve according to table 1.

If we change parameter t with respect to table 1 and assume that t is a time, than we receive time-dependent curve. It will be denoted by Ω. While $t \in [0, t_1]$ a protuberant spherical region $\mathbb{S}$ is formed from line $u = \lambda_0$.

t	$t<0$	$t\in[0,t_1]$	$t\in(t_1,t_2)$	$t\in[t_2,t_3]$	$t>t_3$
$\mu(t)$	0	$\tilde{\mu}(t)$	μ_1	μ_1	μ_1
$\nu(t)$	λ_0	$\frac{\lambda_0^2}{\sqrt{\lambda_0^2+\mu^2}}$	$\tilde{\nu}(t)$	$\frac{k\lambda^2}{\sqrt{\mu_1^2-\lambda^2}}$	0
$\lambda(t)$	λ_0	λ_0	λ_0	$\tilde{\lambda}(t)$	0

Table 1: Parameters λ, μ and ν dependent from t.

As soon as $t\in(t_1,t_2)$ a throat is created on formed protuberant region. When $t\in[t_2,t_3]$ the throat is constricted to a point. At the completion of process the sphere will separate after the throat will have constricted to a point.

3 3-dimensional surface

If we have rotated $\Omega(t)$ at every fixed point t, then we obtain a non-stationary three-dimensional surface $\Sigma(t)$. At every point t, it is described by surfaces $\mathcal{H}(x,y,z,u;t)=0$, if $0\le u\le\nu(t)$, and $\mathcal{S}^3(x,y,z,u;t)=0$, if $u>\nu(t)$, joined together on some 2-dimensional surface $u=\nu(t)$. Let us introduce two three-dimensional non-stationary metrics on surfaces $\mathcal{H}$ and $\mathcal{S}^3$ at every moment t.

A first metric tensor is determined on the spherical part $\mathcal{S}^3$ of surface Σ. We have

$$ds^2=R^2\cos^2\eta\left(\cos^2\psi d\varphi^2+d\psi^2\right)+R^2d\eta^2, \tag{3}$$

$$\varphi\in[0,2\pi],\quad \psi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right],\quad \eta\in\left(\arcsin\alpha,\frac{\pi}{2}\right], \tag{4}$$

where

$$\alpha=\frac{\lambda^8-(\mu^2-\lambda^2)^2\nu^4}{4R\mu^2\lambda^2\nu^3},$$

R was defined by (2). The boundary of the surface $\mathcal{S}^3$ is given by formula $\eta=\arcsin\alpha$.

A second metric tensor $\gamma_{\alpha\beta}(\mathcal{H})$ is determined on the hyperbolic part $\mathcal{H}$ of the surface Σ. Let

$$u^{\pm}(r;t)=\frac{\lambda\mu}{\mu^2-\lambda^2}\left(r\pm\sqrt{r^2-r_0^2}\right), \tag{5}$$

$$u_0(t)=\frac{\lambda^2}{\sqrt{\mu^2-\lambda^2}},\qquad r_0(t)=\frac{\lambda}{\mu}\sqrt{\mu^2-\lambda^2} \tag{6}$$

and

$$r_b(t)=\frac{\lambda^4-(\lambda^2-\mu^2)\nu^2}{2\mu\lambda\nu}. \tag{7}$$

Then $u(r)$ is determined by equations

$$u=\frac{\lambda^2}{2r},\qquad \text{if}\ \left\{\begin{array}{l} r\in[r_b,+\infty),\\ t=t',\end{array}\right. \tag{8}$$

$$u=u^-(r;t),\qquad \text{if}\ \left\{\begin{array}{l} r\in[r_b,+\infty),\\ t\in[0,t')\cup(t',t^*],\end{array}\right. \tag{9}$$

$$u=u^-(r;t),\qquad \text{if}\ \left\{\begin{array}{l} r\in[r_0,+\infty),\\ t\in(t^*,t^3],\end{array}\right. \tag{10}$$

$$u=u^+(r;t),\qquad \text{if}\ \left\{\begin{array}{l} r\in(r_0,r_b],\\ t\in(t^*,t^3],\end{array}\right. \tag{11}$$

Thus, we receive that in moment $t\in[0,t^*]$ a three-dimensional metric tensor on $\mathcal{H}(t)$ corresponds to

$$ds^2=r^2\cos^2\psi d\varphi^2+r^2d\psi^2+\left(1+\left((u^-)'_r\right)^2\right)dr^2,$$

$$\varphi\in[0,2\pi],\quad \psi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right],\quad r\in[r_b,+\infty).$$

And in every point $t\in(t^*,t_3]$ a three-dimensional metric tensor on $\mathcal{H}(t)$ is two three-dimensional metrics pasted together on the two-dimensional surface $r=r_0$. One of them is

$$ds^2=r^2\cos^2\psi d\varphi^2+r^2d\psi^2+\left(1+\left((u^-)'_r\right)^2\right)dr^2,$$

$$\varphi\in[0,2\pi],\quad \psi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right],\quad r\in[r_0,+\infty)$$

and another is

$$ds^2=r^2\cos^2\psi d\varphi^2+r^2d\psi^2+\left(1+\left((u^+)'_r\right)^2\right)dr^2,$$

$$\varphi\in[0,2\pi],\quad \psi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right],\quad r\in(r_0,r_b).$$

Without any restrictions we can introduce an united note for every considering metric:

$$ds^2=r^2\cos^2\psi d\varphi^2+r^2d\psi^2+\left(1+(u'_r)^2\right)dr^2, \tag{12}$$

$$\varphi\in[0,2\pi],\quad \psi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]. \tag{13}$$

The variable r is changed with respect to equations (8 – 11). Herewith u'_r is a derivative of $u(r;t)$ with respect to r. The boundary of the surface $\mathcal{H}$ is given by formula $r=r_b$.

At the fixed parametric point t a metric tensor on the surface $\Sigma(t)$ may be receive by pasting together the metric on $\mathcal{S}^3(t)$ and the metric on $\mathcal{H}(t)$. These metrics is identified on the two-dimensional surface.

4 Four-dimensional metric tensor

Now we can construct a four-dimensional metric tensor g_{ik} on the surface Σ by using Gaussian normal coordinates. There are two expression

$$ds^2 = dt^2 - r^2 d\Omega^2 - \left(1 + (u_r')^2\right) dr^2, \tag{14}$$

$$ds^2 = dt^2 - R^2 \cos^2 \eta d\Omega^2 - R^2 d\eta^2. \tag{15}$$

We used denote $d\Omega^2 = \cos^2 \psi d\varphi^2 - d\psi^2$.

If $t \in [0, t^*]$ then we paste together (15) and (14), where $u = u^-$ and $r \in [r_b, +\infty)$. If $t \in (t^*, t^3]$ then three four-dimensional metric tensors are pasted together. The fist metric is (14) with $u = u^-$ and $r \in [r_0, +\infty)$, the second metric is (14) with $u = u^+$ and $r \in (r_0, r_b)$, and the third metric is (15). The metric tensor (15) describes created baby-universe. If we find the energy-momentum tensor of whole spacetime, then we receive the violation of energy condition in some points. This violation exist in the region $t \in (t^*, t^3]$ and corresponds to negative extrinsic curvature of the hyperbolic part of the surface Σ.

5 Conclusion

We construct metric tensor of spacetime at which the baby-universe is created. Further we can construct a transient intra-universe wormhole with closed timelike curve and without it. The energy conditions are violated in the receiving spacetime. At first moment we have a flat spacetime $ds^2 = dt^2 - r^2 d\Omega^2 - dr^2$, at the end there are a baby-universe and a parent-universe pasted together in some point. At the end the parent-universe becomes flat again.

References

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СОЗДАНИЕ ДОЧЕРНЕЙ ВСЕЛЕННОЙ

Рассматривается метрика 4-мерного пространства-времени, которая описывает создание и отрыв сферы S^3 из изначально плоского пространства-времени. Данный отрыв части 3-мерного пространства можно рассматривать как создание дочерней вселенной. Приведенная метрика может быть использована для построения 4-мерной кротовой норы, топологическая структура которой не эквивалентна $R \times \Sigma$, где Σ – пространственно-подобная гиперповерхность нетривиальной топологии.

Ключевые слова: *кротовая нора, отрыв шара.*

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