SOLVING GENERALIZED FUZZY LEAST COST PATH PROBLEM OF SUPPLY CHAIN NETWORK

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Abstract

Optimal route selection for delivering product is the major concern for organizations related to supply chain management. The choice of route is crucial as it has a big impact on an organization's finances. In this research, an optimum solution with inaccurate and hazy parameters to a fuzzy least cost route issue is presented. Costs can be represented by time, distance or other criteria that could represent edge weights and these are defined by the user. In this paper we are using term cost as activity time. More specifically, the cost value is taken as Generalized hexagonal fuzzy numbers. The paper discusses optimal route selection problem to reduce distance-driven costs. By using ranking method optimal cost value obtained in form of crisp numbers. Also, for the validation of our result and obtained optimal cost in form of fuzzy number, we use fuzzy dynamic programming. We obtain an improved result using our ranking algorithm. Additionally, a comparison is provided. A numerical example for comparison analysis with previous publications is provided, utilising appropriate graphical layout and tables, to elucidate both approaches.

Keywords: Dynamic programming, Generalized hexagonal fuzzy numbers, Supply chain network, Fuzzy least cost path, Ranking Function.

1. Introduction

Managing supply chains with an integrated approach is essential for effectively planning and supervising the movement of products from suppliers to end users. A supply chain is an intricate web of interrelated business networks that includes a number of different organizations, such as distributors, suppliers, manufacturers, warehouses, and shipping companies. The goal is to guarantee that goods flow smoothly through the manufacturing, shipping, and distribution phases, arriving at consumers' locations on schedule and within budget. Modern organizations depend on their supply chains to succeed because they have a direct impact on the profitability, customer satisfaction, and operational efficiency of the organization. For example, a well-managed supply chain can boost product availability, cut down on delays, and lower production costs, all of which can raise sales. However, delays, lost money, and strained client relations can result from supply chain disruptions. Supply chain management has a big impact on corporate success, which is why experts and researchers are interested in learning more about it and improving it. Creating plans that streamline supply chain operations from inventory control and demand forecasting to customer service and transportation scheduling is the main goal.

Cost-effectiveness plays a vital role in today's business environment through process optimization, waste reduction, and better resource utilisation, effective supply chain networks contribute to cost savings. Improved customer satisfaction and loyalty are the result of responsive and dependable supply chain networks, which guarantee prompt delivery, product availability, and high-quality services. Due to shifting market conditions, uncertainity in lead times, and changing demand, supply chain networks are by their very nature unpredictable. Supply chain managers may create more durable and dependable solutions by modelling and optimising distribution networks, inventory flows, and transportation routes under erratic circumstances with FLCPP.

The goal of the least-cost path problem is to find the least expensive way or route between a given starting point and destination. Generally, least-cost path problems are represented using graphs. A set of vertices and edges makes up a mathematical structure called a graph. Vertices are points, while edges are the connections between pairs of vertices. These edges allow movement between vertices. Graphs are either directed or undirected depending on whether movement is allowed in both directions or only in one. If just one direction is permitted for movement along a graph's edges, then that graph is said to be directed. On the other hand, if movement is allowed along the edges in both directions, the graph is categorized as undirected.

A graph's edges are typically given weights in the least-cost path issue, which indicate how much it costs to traverse each edge. The objective is to determine the path that minimizes the overall cost between the beginning and destination vertices. Numerous industries, including logistics, transportation, and communication networks, face these issues. The least-cost path problem is typically solved using algorithms such as Dijkstra's and Bellman-Ford, which identify the shortest path between two places while minimizing the overall cost by allocating weights to the graph's edges.

The traditional least-cost path issue is extended by the fuzzy least-cost path problem (FLCPP), which aims to determine the best cost-effective path between two nodes in a network while taking uncertainty and imprecision into account. Fuzzy set theory is used in FLCPP to model uncertainty, in contrast to the traditional least-cost path problem, which assigns precise values to the costs of traversing arcs or nodes. Fuzzy numbers are employed in this framework to represent the costs, which reflect real-world unpredictability like variable trip charges, irregular travel schedules, or imprecise user preferences. A more flexible and reliable method of modeling complex scenarios is provided by fuzzy numbers, which are inaccurate but realistic representations of these uncertainties. Taking into account the inherent uncertainties in the system, FLCPP seeks to identify the fuzzy path with the maximum degree of satisfaction or the lowest predicted cost. In contrast, standard approaches miss the subtleties of uncertain settings since they depend on stable values. With FLCPP, pathways must be evaluated not only on direct costs but also on decreasing overall uncertainty while satisfying the decision-maker's preferences, which makes the decision-making process more dynamic. The ultimate resolution is to offer the optimal equilibrium between economical viability and minimizing ambiguity. Because unforeseen conditions frequently develop in these domains where flexible decision-making is essential for optimization FLCPP finds applications in a variety of fields, including supply chain management, transportation, and telecommunications.

In supply chain operations, efficient inventory management is essential for reducing holding costs and adjusting stock levels to changing demand. Fuzzy least cost path problem (FLCPP): This sophisticated method of supply chain optimization takes into account a variety of uncertainties, including inaccurate demand estimates, erratic lead times from suppliers, and variable costs associated with retaining inventories. Businesses can use FLCPP to optimize distribution routes and make better decisions about inventory replenishment, which will ultimately save costs and increase efficiency. One significant area of supply chain spending that FLCPP can help with is transportation. Businesses can determine the most cost-effective transportation routes with the use of FLCPP, which accounts for uncontrollable factors like weather, traffic, and fluctuations in fuel prices. As a result, transportation planning becomes more economical and effective, which is essential to sustaining profitability. In general, FLCPP gives companies a competitive edge and boosts productivity in a volatile market by helping them better handle uncertainty in supply chain operations. Businesses can successfully handle difficult supply chain problems by implementing fuzzy logic-based optimization techniques like FLCPP, which promotes long-term growth, better resource allocation, and higher financial gains. Businesses can overcome uncertainty and optimize their operations in a global market that is highly competitive by using this strategic strategy.

Supply chain networks are complex networks of linked companies, resources, activities, and technology that make it easier to produce, distribute, and deliver goods and services from suppliers to end users. Suppliers who offer the necessary parts, raw materials, or services to enable manufacturing are the first in the process. These inputs are subsequently used by manufacturers to create the finished goods. Distributors handle the handling of the products' storage, shipping, and delivery to retailers after manufacturing is finished. Retailers offer the products to final customers through a variety of tactics include direct marketing, internet platforms, and physical storefronts. Eventually, people or businesses use or consume the products or services. From the first stage of production to the last stage of consumption, every link in this chain is essential to the smooth flow of products. Businesses are able to meet customer needs while optimizing costs, time, and resources thanks to this networked system.

In conclusion, supply chain networks are critical to modern businesses' ability to satisfy customer needs, keep costs under control, and effectively allocate resources. By streamlining processes and boosting responsiveness, efficient supply chain management gives businesses a competitive edge in today's linked and dynamic global marketplaces. Businesses can attain operational excellence, which guarantees timely delivery of goods and services while cutting costs, by optimizing the manufacturing, distribution, and delivery procedures. An effective supply chain also fosters business sustainability and growth, generating long-term value for all parties involved. In the end, a company's capacity to succeed and adapt in a more competitive business environment ultimately depends on its supply chains being well-managed.

Motivation of this Study

The path with the lowest cost for individuals going between a source and a destination is referred to as the least cost path. In a directed acyclic network, exact values are typically used as weights on the edges. But a number of circumstances including shifting traffic patterns, unfavorable weather patterns, and natural disasters make it challenging to determine these weights precisely. Under such circumstances, traditional techniques might not be sufficient to capture the uncertainties prevalent in real-world scenarios. This emphasizes the need for alternative methods that can account for imprecision and fluctuation in the weights, like fuzzy set theory. Decision-makers can improve route optimization by more skillfully assessing the most cost-effective routes under dynamic conditions by including these uncertainties.

Consider a scenario in which a transportation company is tasked with determining the least-cost route for delivering goods across various locations. The total cost for any given route is influenced by uncertain factors such as fluctuating fuel prices, varying toll fees, and unpredictable road conditions. Traditional least-cost path algorithms typically rely on precise, fixed values for these factors, which may not adequately reflect real-world conditions. For instance, fuel prices can change frequently and vary significantly from one state to another. Within the state of Himachal Pradesh, for example, the petrol prices differ across various cities: Una has a price of 93.20 rupees, Hamirpur 93.68 rupees, Solan 94.20 rupees, Nahan 94.47 rupees, Chamba 95.00 rupees, and Kullu 95.20 rupees. Given this variability, the fuel cost can be effectively represented using a hexagonal fuzzy number, specifically $\tilde{F} = (93, 93.50, 94, 94.50, 95, 95.50)$. This approach allows for a more flexible representation of fuel costs, accounting for the inherent uncertainties present in the transportation network.

Objective

The objective of this paper is to apply an existing ranking methodology to solve the fuzzy least-cost route selection problem in supply chain management, where uncertainties like time and cost are critical factors. By using generalized hexagonal fuzzy numbers to represent these uncertain parameters, the study aims to better capture the imprecision inherent in real-world transportation scenarios. The goal is to convert fuzzy values into crisp numbers, enabling more accurate and informed decision-making for optimal route selection. The method is validated through fuzzy dynamic programming, ensuring that the results are robust and applicable across various uncertain conditions. A comparative analysis with previous methods is provided to highlight the effectiveness of the approach. Additionally, a numerical example demonstrates how this model can be practically applied, offering insights for organizations seeking to minimize costs and manage uncertainty in their supply chain operations.

Advantages of our result

1. A comparison is provided with existing approaches, highlighting the effectiveness of the proposed method.
2. The use of a ranking function allows decision-makers the flexibility to incorporate multiple criteria, enabling more informed decision-making.
3. Fuzzy ranking handles uncertainties and imprecise data better than deterministic approaches, enhancing the robustness of the solution against variations and errors in the input parameters.
4. To validate the results, a previously established dynamic programming technique has been applied.
5. The results of fuzzy ranking are often easier to interpret, providing insights into the relative importance of different factors, thus supporting transparent decision-making.
6. The methodology used in this paper is straightforward and can be easily applied in practice.
7. The outcome is represented in the form of generalized hexagonal fuzzy numbers, which allow for comparison through various defuzzification methods to obtain crisp values.

Contributions

- Introduced the use of generalized hexagonal fuzzy numbers to represent costs in route selection, enhancing flexibility and accuracy in cost estimation under uncertainty.
- Applied fuzzy dynamic programming to validate the results, ensuring the reliability and applicability of the optimal solutions in dynamic and uncertain environments.
- Conducted a comprehensive comparative analysis with previous publications, providing insights into the effectiveness of the proposed method through graphical representations and tables.
- Utilized numerical examples to illustrate the practical implications of the proposed approach

The format of this document is as follows: We have provided some fundamental definitions and introductions in Section 2. The generalised hexagonal fuzzy number ranking algorithm is defined in Section 3. The least-cost path problem and the ranking method's solution are described in Section 4 with the conventional fuzzy approach method for addressing dynamic programming. Furthermore, a numerical case was resolved using each approach. Section 5 provide the outcome. The comparison is presented in Sections 6 respectively. Section 7 provides the conclusion and future research scope.

Literature Review

Initial studies (1960s-1980s), Initial mathematical models for inventory control and transportation were the focus of supply chain optimisation research. To maximise choices about production and distribution, linear programming techniques were frequently applied. In 1970 Bellman et al. [1] intoduced fuzzy dynamic programming. In which the concept of fuzzy set theory was used in decision making problems.

Expanding the Scope (1990s): During this decade, there was a change in the focus of supply chain optimisation to include more facilities, decision points, and tiers. More complex models, such as mixed-integer programming and heuristic methods for handling larger-scale issues, were introduced at this time. Later, the applications of fuzzy dynamic programming have given by Kacprzyk. With the help of this Hussien et al. [2] solved multiple criteria resources allocation problem. For multistage problem Baldwin et al. [3] have worked on fuzzy dynamic programming.

The 2000s saw the integration of information technology: As the internet and information technology grew in popularity, supply chain optimisation studies began to use real-time data and sophisticated analytics. During this time, supply networks were encouraged to be transparent, cooperative, and responsive.

Within the framework of supply chain optimisation, sustainability and risk management have received increasing attention in the 2010s. Environmental factors, such reducing carbon footprints and implementing green logistics, have been included by researchers into optimisation models. The mitigation of risks related to interruptions, such as natural catastrophes and geopolitical events, was also emphasised.

Industry 4.0 and digitalization (2015-present): The introduction of Industry 4.0 technology and the digitization of supply networks define the contemporary age. These days, big data analytics, blockchain, artificial intelligence, and the Internet of Things (IoT) are all included in optimisation research. These technologies give supply chain operations more flexibility, more visibility, and real-time decision-making capabilities. Supply chain optimisation research has developed across these phases in response to novel problems and chances brought about by modifications in company practices, technological breakthroughs, and evolving consumer expectations. A article on addressing a fuzzy optimum subdivision problem utilising the fuzzy least cost route problem by employing the generalised trapezoidal number has been proposed by Nagalakshmi et al. [4].

Chang and Zadeh established the idea of fuzzy mapping and control. A multitude of techniques have been presented for ranking fuzzy numbers ([5], [6], [7]). Additionally, a crucial part of decision-making is the ranking of fuzzy numbers. To defuzzify the fuzzy numbers, several efforts have been undertaken ([8], [9], [10], [11]). According to Chen et al.[12], for regular fuzzy numbers, there is no requirement for the membership function. The idea of generalised fuzzy numbers was first out by him. In order to rank fuzzy numbers, Yager [13] employed the centroids technique. Using the modes, rank, divergence, and spread that Rajarajeshwari and Sudha [14] presented, we may order generalised hexagonal fuzzy numbers.

Many studies have taken into account the multi-objective aspect of FLCPP by include many competing elements in the optimisation model, including cost, travel time, and dependability. Hybrid techniques, which integrate many approaches to enhance the quality and effectiveness of solutions, have been suggested in numerous research. To incorporate the benefits of both techniques, a hybrid algorithm may, for instance, combine a metaheuristic optimisation algorithm with a fuzzy logic-based decision-making module. Within supply chain networks, where uncertainty and imprecision are significant variables, there are several real-world applications of the fuzzy least cost path problem (FLCPP).

Supply chain logistics experts may utilise FLCPP to optimise transportation routes by accounting for irregular factors including weather fluctuations, traffic congestion, and bad road conditions. Atkinson et al. [15] provided a multi-criteria analysis-based least-cost-path technique for the route of an all-weather road. Because fuzzy logic enables the model to adapt to changing conditions and provide dependable route recommendations, it lowers transportation costs and delivery delays.

Numerous research projects have been carried out on the fuzzy shortest path problem (FSPP). Within two locations, there are several ways and routes. When making decisions on the least amount of time and money to travel, the most reliable route, the least amount of traffic, etc., this information is crucial. Many real-world issues, such as scheduling [16, 17] and telecommunication communications [18, 19], were modelled by decision makers using SPP. The shortest path must be found by decision makers, who are viewed as a graph, in order to solve this optimisation challenge. For instance, the primary goal of the fire stations is to get the fire van to the house in their service area as quickly as possible. The lowest damaged path for the fastest evacuation is a highly valuable tool for disaster management.

The fuzzy shortest route problem was first proposed by Dubois and Prade in [20]. A fuzzy shortest route technique, based on the classic Ford Moore Bellman algorithm, was presented by them. In their study, they used the concept of a path's criticality.When Chanas and Kamburowski in [21] suggested utilising fuzzy preference relationship to find the shortest path and included it in their SPP approach, they introduced the concept of fuzzy preference relationship. Each edge was represented by an integer number between a given higher integer number and 1 by the authors of [22].

Taking into consideration the dynamic programming approach, they created an algorithm that selects a path depending on the membership grade ascertained by an expert. Linet al. [23] considered an edge to be the most significant (vital) edge in the graph if its removal increased the shortest path maximum. They have produced a fuzzy membership function for the shortest path by applying a linear programming approach. An algorithmic method has been suggested by them to identify the single most important edge in a fuzzy network.

Takahashi et al. introduced the fuzzy arc length SPP in [24], expanding upon Okada [25]'s originally proposed technique. A large fuzzy graph's estimated shortest path may be found using a genetic method, according to the authors. Interval and triangle fuzzy numbers were used by Nayeem and Pal [26] to define a graph's edge weights. Both kinds of numbers can be handled by their suggested approach, which can also solve the fuzzy shortest route issue.

An algorithmic method for resolving the fuzzy shortest route issue was presented by Hernandes et al. [27]. The weight of each edge has been represented by the authors using a triangular fuzzy number. Mahdavi and colleagues [28] introduced a dynamic programming method for solving the fuzzy shortest path chain issue. For their algorithm, they proposed a ranking system, which can assist in preventing the set of shortest pathways from being created because there may be a large number of shortest paths overall from a large fuzzy graph. Even for a specialist, selecting the precise shortest path might be quite challenging. For the purpose of solving the shortest path issue in an unpredictable environment, Deng et al. have given an expanded traditional Dijkstra's approach in [29]. By employing trapezoidal fuzzy numbers, the authors have expressed the fuzzy network's edge weights.They used the graded mean integration technique of fuzzy numbers in their approach to calculate the fuzzy route's length and contrast the fuzzy path distances between two different fuzzy pathways.

Hassanzadeh et al. suggested a method in [30] using fuzzy edge weights on a fuzzy graph, to determine the shortest path. The authors used an addition procedure based on α cut to determine the path's length. Using a least squares method for their addition operation, they suggest building an approximation fuzzy membership function for the proper addition process. A genetic strategy is also proposed to solve the fuzzy SPP and deal with the challenge of adding fuzzy numbers for a large-scale fuzzy network.

FLCPP may be used to address inventory routing problems while replenishing inventory levels across several locations while taking erratic demand patterns and supply disruptions into consideration. Routing strategies that balance inventory costs and service levels can be produced via an optimisation model that accounts for fuzzy demand estimates and fuzzy trip lengths.Fuzzy least cost path algorithms, which identify reliable supply chain routes with low impact from uncertainty and disturbances, can be used to reduce supply chain risk. Using probabilistic models and fuzzy risk assessments, decision-makers may proactively predict and avoid risks related to supplier failures, natural disasters, and delays in transportation.

Because demand patterns and market circumstances are unpredictable, FLCPP's analysis of possible configurations can assist in making judgements regarding the placement of facilities and network architectures. By accounting for factors like fuzzy location preferences, transportation costs, and projected market demand, businesses may strategically place their facilities and build supply chain networks that are robust and flexible to changing customer wants. These real-world applications demonstrate FLCPP's versatility in addressing a range of supply chain network optimisation problems, including inventory management, transportation, and network design in addition to risk reduction. Businesses may make educated decisions that improve the adaptability, efficiency, and resilience of their supply chains in the face of uncertainty by utilising fuzzy logic and optimisation techniques.

2. Defination & Preliminaries

2.1. Fuzzy Set

A set $\tilde{I}$ defined as $\tilde{I} = \{(u, \mu_{\tilde{I}}(u)) : x \in I, \mu_{\tilde{I}}(u) \in [0,1]\}$, where $\mu_{\tilde{I}}(u)$ is membership function of $\tilde{I}$, is called a fuzzy set.

2.2. Hexagonal Fuzzy Number

Let $\tilde{I_h} = (i_1, i_2, i_3, i_4, i_5, i_6)$, be a fuzzy set defined on $\Re = (-\infty, \infty)$, which is called a hexagonal fuzzy number if the membership function of $\tilde{I_h}$ is given by

$$\mu_{\tilde{I_h}}(t) = \begin{cases} \frac{1}{2}(\frac{t-i_1}{i_2-i_1}), & \text{if } \; i_1 \leq t \leq i_2 \\ \left(\frac{1}{2} + \frac{t-i_2}{2(i_3-i_2)}\right), & \text{if } \; i_2 \leq t \leq i_3 \\ 1, & \text{if } \; i_3 \leq t \leq i_4 \\ \left(1 - \frac{i_4-t}{2(i_5-i_4)}\right), & \text{if } \; i_4 \leq t \leq i_5 \\ \left(\frac{i_6-t}{2(i_6-i_5)}\right), & \text{if } \; i_5 \leq t \leq i_6 \\ 0, & \textit{otherwise} \end{cases}$$

2.3. Generalized Hexagonal Fuzzy Number

Let $\tilde{I_{gh}} = (i_1, i_2, i_3, i_4, i_5, i_6; w)$ be a fuzzy set defined on $\Re = (-\infty, \infty)$ and w is its maximum degree of membership function. The membership function of $\tilde{I_{gh}}$ is given by

$$\mu_{\tilde{I_{gh}}}(t) = \begin{cases} 0, & \text{if } \; t \leq i_1 \\ \frac{w}{2}(\frac{t-i_1}{i_2-i_1}), & \text{if } \; i_1 \leq t \leq i_2 \\ \frac{w}{2} + \frac{w}{2}(\frac{t-i_2}{i_3-i_2}), & \text{if } \; i_2 \leq t \leq i_3 \\ w, & \text{if } \; i_3 \leq t \leq i_4 \\ 1 - \frac{w}{2}\frac{i_4-t}{i_5-i_4}, & \text{if } \; i_4 \leq t \leq i_5 \\ \frac{w}{2}(\frac{i_6-t}{i_6-i_5}), & \text{if } \; i_5 \leq t \leq i_6 \\ 0, & \text{if } \; t \geq i_6 \end{cases}$$

2.4. Arithmetic operation on GHFN (Generalized Hexagonal Fuzzy Number):

Let $\tilde{I}_{gh_1} = (i_1, i_2, i_3, i_4, i_5, i_6; w_1)$ and $\tilde{I}_{gh_2} = (j_1, j_2, j_3, j_4, j_5, j_6; w_2)$ be two GHFNs then

1. **Equality of two GHFNs:**

   $\tilde{I}_{gh_1} = \tilde{I}_{gh_2}$ iff $i_1 = j_1, i_2 = j_2, i_3 = j_3, i_4 = j_5, i_5 = j_5, i_6 = j_6$ and $w_1 = w_2$

2. **Addition of two GHFNs:**

   $\tilde{I}_{gh_1} + \tilde{I}_{gh_2}$=$(i_1 + j_1, i_2 + j_2, i_3 + j_3, i_4 + j_4, i_5 + j_5, i_6 + j_6; w)$ where $w = min(w_1, w_2)$

3. **Subtraction of two GHFNs:**

   $\tilde{I}_{gh_1} - \tilde{I}_{gh_2}$=$(i_1 - j_1, i_2 - j_2, i_3 - j_3, i_4 - j_4, i_5 - j_5, i_6 - j_6; w)$ where $w = min(w_1, w_2)$

3. Ranking of Generalized Hexagonal Fuzzy Numbers [31]

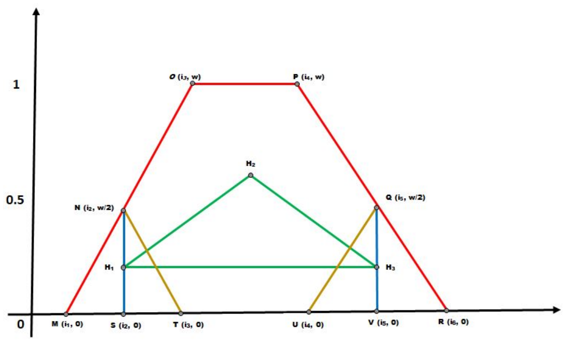

Figure 1: *Generalized hexagonal fuzzy numbers. Dividing hexagonal into three plain figure and taking centeriod say H_1, H_2, H_3 of these three figures will here define the ranking function.*

In Figure 1, the hexagon's balance point is thought to represent the centroid of a fuzzy number. The hexagonal may be divided into three planar forms. The triangle MNT, hexagon $OPQVTN$, and triangle VQR are the three figures, in that order. The generalised hexagonal fuzzy number ranking is defined by using the circumcenter of the centroids of these three planar figures as the reference point. Let H_1, H_2 and H_3 be the centroid of the corresponding three planar figures.

The centroid of the three plane figures is

$$H_1 = (\frac{i_1 + i_2 + i_3}{3}, \frac{w}{6}), \qquad H_2 = (\frac{i_2 + 2i_3 + 2i_4 + i_5}{6}, \frac{w}{2}), \qquad H_3 = (\frac{i_4 + i_5 + i_6}{3}, \frac{w}{6})$$

The line H_1, H_3 has the equation y=$dfracw6$, and H_2 is not on the line H_1, H_3. As a result, H_1, H_2, and H_3 form a non-collinear triangle.
The generalised hexagonal fuzzy number $\tilde{I_{gh}} = (i_1, i_2, i_3, i_4, i_5, i_6; w)$, and its ranking function $R(\tilde{f_{gh}})$ map the set of all fuzzy numbers to $\Re = (-\infty, \infty)$.

$$R(\tilde{f_{gh}}) = (x_0)(y_0) = (\frac{2i_1 + 3i_2 + 4i_3 + 4i_4 + 3i_5 + 2i_6}{18} \times \frac{5w}{18}) \qquad ....(1)$$

4. Least cost path Problem [4]

The least-cost path problem is a critical aspect of transportation and logistics that focuses on allocating resources efficiently along various routes to achieve the lowest possible cost between a starting point and a destination. The objective is to identify the most economical path from the initial node (Node 1) to the target node (Node n). In this context, each edge connecting the nodes is weighted to reflect the associated costs, which may include factors such as travel time, fuel expenses, and toll fees. Each city within the network is represented as a node, creating a framework for analyzing potential routes.

Determining the optimal route to connect these locations is essential for effective decision-making, as it directly impacts operational efficiency and cost management. By concentrating on minimizing costs, this approach not only enhances the overall performance of transportation systems but also aids organizations in making informed choices regarding resource allocation. Additionally, addressing the least-cost path problem contributes to the development of more sustainable transportation solutions, as it encourages the utilization of routes that optimize both time and financial resources. Ultimately, a thorough understanding of this problem is vital for improving supply chain management and enhancing service delivery in the logistics sector.

In addressing the problem at hand, we recognize that it unfolds in distinct phases, with each level requiring a decision to be made among several available options. At step ①, the decision-maker must select one of the three potential pathways: (1,2), (1,3), or (1,4). Each of these options presents different implications for the overall route and associated costs. The ultimate goal is to identify the optimal policy, which comprises a series of interconnected pathways or routes that effectively link the starting point ① to the destination ⓝ. This optimal policy will not only minimize costs but also accommodate the inherent uncertainties in the decision-making process. By systematically evaluating each pathway at every phase, the decision-maker can navigate through the complexities of the problem, ensuring that the selected route aligns with the overarching objectives of efficiency and cost-effectiveness. The following numerical example illustrates this decision-making process, providing clarity on how these pathways interact and contribute to the overall solution.

The problem can be approached through either forward or backward recursive equations, each providing a unique pathway to arrive at the solution. In forward recursion, the process begins at the initial state and progresses sequentially toward the final state. This method allows for a step-by-step evaluation of each decision point as the solution unfolds, ultimately leading to the desired outcome. Conversely, backward recursion starts at the end of the process and works its way back to the beginning. This approach is particularly useful when the final outcomes are known, allowing for the determination of optimal decisions by tracing back through the various pathways. By evaluating the potential consequences of each decision in reverse order, the decision-maker can identify the best route to achieve the final objectives. Both methods have their advantages, depending on the specific context of the problem. Forward recursion may be more intuitive for problems where the sequence of actions is clear and linear, while backward recursion can be advantageous in complex scenarios where understanding the end conditions is crucial. Ultimately, the choice between forward and backward recursion will depend on the nature of the problem and the preferences of the decision-maker.

4.1. Fuzzy Least Cost Path Problem

In a fuzzy environment, the problem at hand is addressed by considering activity time as a generalized hexagonal fuzzy number. This approach allows for a more accurate representation of uncertainties inherent in real-world scenarios, facilitating better decision-making throughout the optimization process.

4.2. Procedure of solution using Ranking method

To address the least-cost path problem, the following steps should be undertaken:

Step 1: We analyze the acyclic network, utilizing hexagonal fuzzy numbers to calculate the edge weights for improved decision-making.

Step 2: Find the value of (x_0) & (y_0).

Step 3: Defuzzify the edge weights by using ranking method mentioned in section 3.

Step 4: At each stage of the network, systematically calculate the cost value associated with each pathway.

Step 5: Find the total cost value for each path(p_i).

Step 6: Now, to find least, cost consider the minimum cost value obtained in step 5. i.e. suppose $\exists\ n$ path then-

$$\text{Least cost value} = \text{Min}[costvalue(p_1), costvalue(p_2), costvalue(p_3), ...costvalue(p_n)]$$

The cost value of end node is assumme as 0 which is in the last stage.

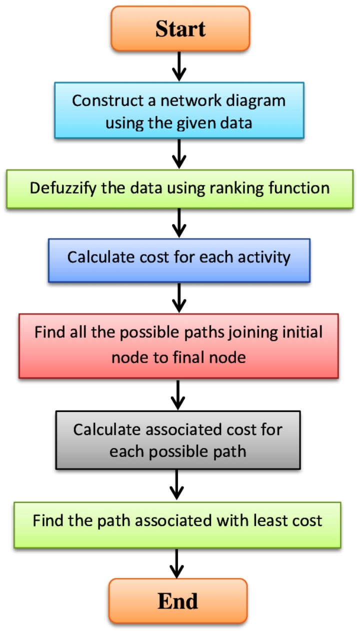

Figure 2: *Flow chart of the method.*

Validation

A potent mathematical method for resolving complicated decision-making issues is dynamic programming (DP), which divides larger problems into smaller, linked subproblems. It is an optimization technique that breaks down a more complex issue into a number of easier-to-manage issues, each of which adds to the final answer. In contrast to conventional mathematical formulations, decision-making process optimization (DP) offers a methodical approach to identify the best possible collection of decisions, rather than imposing a rigid mathematical model. This method provides a flexible framework for handling many different kinds of problems, with individual equations and solutions tailored to the unique circumstances of each one.

Dynamic programming is built around the concept of solving multistage problems, where a complex problem is broken down into several smaller stages, each linked in a specific order. At each stage, a decision needs to be made, and that decision immediately affects what happens next. The goal in these problems is to find a sequence of decisions that leads to the best possible outcome for the entire process. As you make decisions at each stage, the sequence gradually forms, guiding you toward the optimal solution step by step [32].

Dynamic programming (DP) was introduced to solve problems where decision-making factors change over time, which is common in real-world scenarios. It plays a crucial role in optimization, where the goal is to find either the maximum or minimum of a given objective. DP works by breaking down complex problems into smaller, interrelated sub-problems, known as phases, making them easier to solve. Fuzzy set theory, introduced by Zadeh [33], provides an effective way to handle uncertainty and vagueness in such problems. It has become an essential tool for dealing with real-world challenges where data may be imprecise or incomplete. Bellman et al.[1] linked fuzzy set theory to decision-making because the decision-making process often involves uncertainty and ambiguity. By integrating fuzzy set theory with dynamic programming, fuzzy dynamic programming (FDP) emerged as a method for solving optimization problems where parameters are not clearly defined.

In this research, we apply fuzzy dynamic programming to find the least-cost path, where the activity times (or costs) are represented as fuzzy numbers. This approach allows us to handle the uncertainties inherent in real-world logistics and supply chain problems, providing a more flexible and accurate solution to complex decision-making challenges.

Fuzzy Forward and Fuzzy Backward recursive equations

The dynamic programming model include

1. Let s_p be any state, then then it describes a specific city at stage p.
2. The path starting from one stage to the following stage known as decision alternatives.
3. Decision variable from state $p-1$ to p represent by d thus the state changes from s_{p-1} to s_p.
4. From decision $d(s_{p-1}, s_p)$ return will be represented by $f_p(s_p)$.
5. From s_1 to s_p the minimum cost will be denoted by $f_p{}^*(s_p)$.

The Fuzzy Forward recursive equations is

$$\tilde{f}_1(s_1) = \tilde{d}(s_1, s_2)$$
$$\tilde{f}_p(s_p) = Min\{\tilde{d}(s_p, s_{p+1}) + \tilde{f}_{p-1}(s_{p-1})\} \text{ where } p = 2, 3, ..., n$$

The Fuzzy Backward recursive equations is

$$\tilde{f}_n(s_n) = \tilde{d}(s_n, s_{n+1})$$
$$\tilde{f}_p(s_p) = Min\{\tilde{d}(s_p, s_{p+1}) + \tilde{f}_{p+1}(s_{p+1})\} \text{ where } p = n-1, ..., 1$$

Numerical Example

Consider a logistics company responsible for delivering goods from a central warehouse (①) to a retail store (⑧), with several distribution centers in between, namely ②,③,④,⑤, and ⑥. The travel times between these nodes are uncertain, influenced by factors such as traffic conditions and operational delays. To account for this uncertainty, the travel times are represented by Generalized Hexagonal Fuzzy Numbers (GHFNs).

Table 1 presents the fuzzy activity times associated with each activity, reflecting the variability in delivery durations. Figure 3 illustrates the graphical representation of these fuzzy activity times, providing a visual understanding of the distribution and uncertainty in the delivery process. The objective of this analysis is to determine the optimal route time between nodes ① and ⑧, ensuring efficient delivery while accommodating the inherent uncertainties in travel times. Our aim is to analyze different routes and identify the one that minimizes overall delivery time in its logistics operations.

Table 1: *Cost value in terms of Hexagonal fuzzy numbers*

Cost variable	Fuzzy activity duration
C_1: 1-2	(20,25,30,35,40,45)
C_2: 1-3	(11,12,13,14,15,16)
C_3: 1-4	(30,32,35,36,40,42)
C_4: 2-5	(11,15,17,18,20,25)
C_5: 3-6	(11,11,12,12,13,16)
C_6: 4-5	(60,62,65,66,68,70)
C_7: 4-6	(41,43,45,47,49,50)
C_8: 4-7	(2,4,6,8,10,12)
C_9: 5-8	(80,82,87,89,90,95)
C_{10}: 6-8	(52,53,55,55,56,59)
C_{11}: 7-8	(15,17,19,25,28,30)

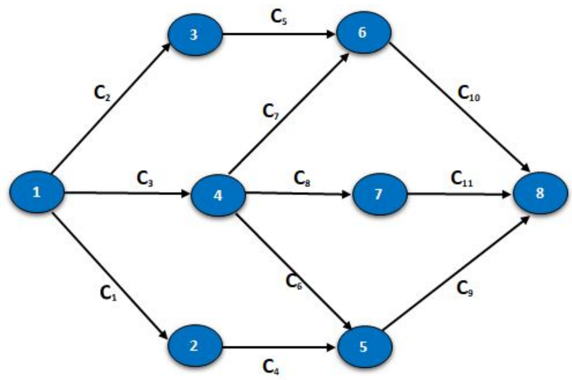

Figure 3: *Acyclic network in terms of hexagonal fuzzy numbers having total 8 events and 11 activities.*

The ranking approach for generalized hexagonal fuzzy numbers, which is discussed in Section 3, is used to transform hexagonal fuzzy costs into crisp values. These expenses are calculated as the cost connecting the two nodes.

Solution:

(i) Using ranking method mentioned in Section 3:

Step 1: Considering the cost variable C_1 between nodes (1, 2), the edge weight associated with this connection is represented as a hexagonal fuzzy number, specifically $(20, 25, 30, 35, 40, 45)$. This representation reflects the uncertainty in the cost estimates for this route.

Step 2: Find the value of (x_0) & (y_0).

(i) For the value of x_0:

$$x_0 = \frac{2i_1 + 3i_2 + 4i_3 + 4i_4 + 3i_5 + 2i_6}{18} = \frac{2(20) + 3(25) + 4(30) + 4(35) + 3(40) + 2(45)}{18} = 32.5$$

(ii) For the value of y_0:

$$y_0 = \frac{5w}{18} = \frac{5(1)}{18} = 0.277777778$$

Step 3: Using the ranking technique outlined in Section 3 to defuzzify the edge weights. This method makes it easier to transform fuzzy values into precise numbers, which improves the ability to compare edge weights and makes route selection decision-making easier.

$$R(\tilde{f_{gh}}) = (x_0)(y_0) = (32.5)(0.277777778) = 9.03$$

Step 4: To calculate the time required for each activity:

Table 2: *Cost value associated with each activity.*

Activity	Fuzzy activity duration	x_0	y_0	$R(\tilde{f_{gh}})$
① → ②	(20,25,30,35,40,45)	32.5	0.277777778	9.03
① → ③	(11,12,13,14,15,16)	13.5	0.277777778	3.75
① → ④	(30,32,35,36,40,42)	35.78	0.277777778	9.94
② → ⑤	(11,15,17,18,20,25)	17.61	0.277777778	4.89
③ → ⑥	(11,11,12,12,13,16)	12.33	0.277777778	3.43
④ → ⑤	(60,62,65,66,68,70)	65.22	0.277777778	18.12
④ → ⑥	(41,43,45,47,49,50)	45.89	0.277777778	12.75
④ → ⑦	(2,4,6,8,10,12)	7.00	0.277777778	1.94
⑤ → ⑧	(80,82,87,89,90,95)	87.22	0.277777778	24.23
⑥ → ⑧	(52,53,55,55,56,59)	54.94	0.277777778	15.26
⑦ → ⑧	(15,17,19,25,28,30)	22.28	0.277777778	6.19

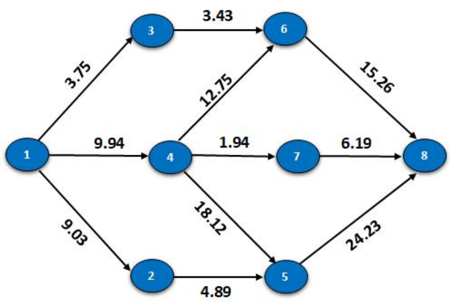

Figure 4: *Acyclic network in crisp form.*

Step 5: To determine the total duration for each path from the starting node ① to the ending node ⑧, we sum the time values associated with the edges along each route. This calculation includes the fuzzy durations for each activity, providing a clear estimate of the total time needed to travel between the two nodes.

Table 3: *Cost associated with each path.*

Path	Associated Cost Value
P_1 : ①→③→⑥→⑧	22.44
P_2 : ①→④→⑥→⑧	37.95
P_3 : ①→④→⑦→⑧	18.07
P_4 : ①→④→⑤→⑧	52.29
P_5 : ①→②→⑤→⑧	38.15

5. Result

In this study, we utilized the ranking function method to determine the least cost path, with Generalized Hexagonal Fuzzy Numbers (GHFNs) representing the uncertain activity times. By using GHFNs, we were able to effectively model various levels of uncertainty and imprecision inherent in the transportation process. This approach is especially useful in real-world scenarios where activity times, such as travel durations or fuel costs, are not fixed but subject to variability and ambiguity. The use of GHFNs allows for a detailed and flexible representation of these uncertainties, ensuring that fluctuations in activity times are accounted for. This method provides a more accurate and comprehensive understanding of the potential costs involved, enhancing decision-making in least-cost path analysis.

To determine the least fuzzy activity time of the path connecting ① to ⑧, we employ the forward procedure outlined in the proposed approach. The fuzzy activity time of the optimal path is computed as:

$$\text{Least cost value= Min } [22.44, 37.95, 18.07, 52.29, 38.15] = 18.07$$

Least cost value route = P_3 : ①→ ④ → ⑦ → ⑧ This value represents the fuzzy duration of the optimal path, taking into account uncertainty and imprecision in the activity times. The path linking ① to ④, ⑦, and ⑧ is determined to be the optimal one. By further applying the fuzzy backward recursive equations, we validate this optimal path, ensuring consistency with the forward approach.

To translate the fuzzy activity time into a more actionable result, we use the ranking function described in Section 3 of the Generalized Hexagonal Fuzzy Method (GHFM). This ranking function converts the fuzzy values into a crisp form, allowing for a precise evaluation of the optimal path. The least crisp activity time for the path from ① to ⑧ is calculated as:

$$\text{Crisp Activity Time} = 18.07$$

The conversion from fuzzy to crisp values provides a concrete measure of the total duration, which aids decision-makers in assessing and comparing potential routes. Furthermore, this result is visually demonstrated in Figure 5, where the least-cost path is depicted with a red dotted line connecting node ① to the destination node ⑧. The optimal path is highlighted, confirming that the path $C_3 \rightarrow C_8 \rightarrow C_{11}$ has the least cost in fuzzy form as $(47, 53, 60, 69, 78, 84)$ and in crisp form as 18.07. The graphical representation supports the findings of the ranking function and enhances the clarity of the optimal route selection.

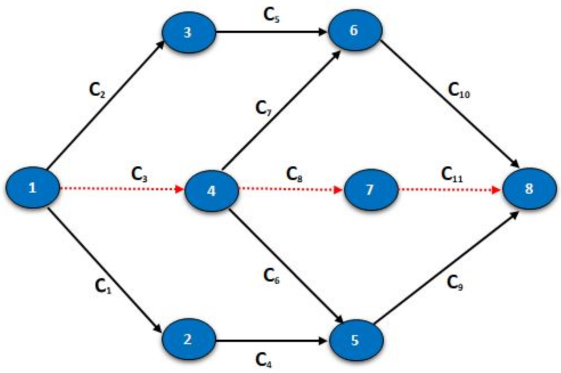

Figure 5: *Least cost value path.*

Validation

To solve the problem of identifying the optimal route for transportation, we break the process down into distinct stages, state variables, and decision variables. We begin by calculating the distances between the cities, starting with the first city, ① $(= x_1)$. From this initial point, we evaluate the distances to the next city, x_2, followed by x_3, and finally x_4. Each decision variable represents the choice of the route taken between consecutive cities. The state variables consist of the current city being analyzed, which helps track progress through the network of cities. By systematically calculating the distances at each stage, we can assess potential routes and determine the one that minimizes overall transportation costs. This approach not only allows for the identification of the best route but also incorporates any uncertainties in the distance data, ultimately facilitating a more effective decision-making process in route selection.

Fuzzy forword recursive equation:

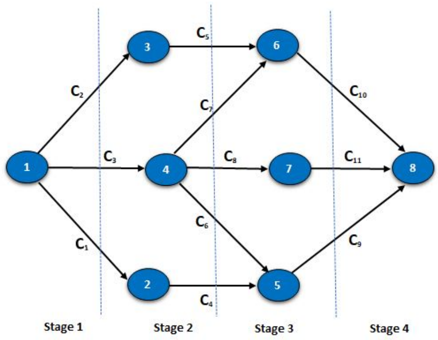

Figure 6: *Stages in network diagram*

Stage 2: In Stage 1, we start by finding the minimum distance from city ① to the cities in Stage 2: $s_2 =$ ②, $s_2 =$ ③, and $s_2 =$ ④. This step helps us understand the potential routes we can take from our starting point. As we move forward, we continue to assess the distances, working our way from city ① all the way to city ⑧. To make this process clearer, table 4 displays the distances necessary to get from Stage 1 to Stage 2. By looking closely at these values, we can pinpoint the most efficient routes and fine-tune our calculations to find the optimal path for our transportation needs.

$$\tilde{f}_2(s_2) = \tilde{d}(s_2, s_1)$$

Table 4: *Stage 2*

s_2	$s_1 = ①$	$\tilde{f}_2(s_2)$	s_1^*
③	$(11, 12, 13, 14, 15, 16)$	$(11, 12, 13, 14, 15, 16)$	①
②	$(20, 25, 30, 35, 40, 45)$	$(20, 25, 30, 35, 40, 45)$	①
④	$(30, 32, 35, 36, 40, 42)$	$(30, 32, 35, 36, 40, 42)$	①

The lowest value among $\tilde{d}(s_2, s_1)$ represents the ideal value of $\tilde{f}_2(s_2)$. In this step, for state $s_2 =$ ④, we find the minimal value to be $(30, 32, 35, 36, 40, 42)$. This result reflects the most favorable outcomes for the distances associated with reaching state ④, providing a clear indication of the optimal route's effectiveness at this stage in our analysis. By identifying this minimal value, we can make more informed decisions as we proceed to the next stages of our route optimization process.

Stage 3: This Stage will follow a similar process to that of Stage 2. We will obtain the next optimal value by adding the minimal value from this Stage to the minimum value identified in Stage 2. By maintaining this consistent approach, we ensure that each stage builds upon the previous calculations, allowing us to systematically refine our route selection. This cumulative method enhances our ability to identify the most efficient path while considering the uncertainties and varying parameters at each step.

$$\tilde{f}_3(s_3) = Min\{\tilde{d}(s_3, s_2) + \tilde{f}_2(s_2)\}$$

Table 5: *Stage 3*

s_3	$s_2 = ③$	$s_2 = ②$	$s_2 = ④$	$\tilde{f}_3(s_3)$	s_2^*
⑥	$(22, 23, 25, 26, 28, 32)$	-	$(71, 75, 80, 83, 89, 92)$	$(22, 23, 25, 26, 28, 32)$	③
⑤	-	$(31, 40, 47, 53, 60, 70)$	$(90, 94, 100, 102, 108, 112)$	$(31, 40, 47, 53, 60, 70)$	②
⑦	–	–	$(32, 36, 41, 44, 50, 54)$	$(32, 36, 41, 44, 50, 54)$	④

Consequently, we achieve $\tilde{f}_3(s_3)$ at its ideal value. For state $s_3 =$ ⑦, we find the associated values in Stage 3 to be $(32, 36, 41, 44, 50, 54)$. This set of values reflects the minimal distances for reaching state ⑦, indicating the optimal routes available at this stage.

Stage 4: Following the same procedure as before, we will add the minimum value from this stage to the minimum value obtained in stage 3 to determine the next best value. This approach ensures that each stage builds on the previous results, helping us refine our route selection as we move forward in the optimization process.

$$\tilde{f}_4(s_4) = Min\{\tilde{d}(s_4, s_3) + \tilde{f}_3(s_3)\}$$

Table 6: *Stage 4*

s_4	$s_3 = ⑥$	$s_3 = ⑤$	$s_3 = ⑦$	$\tilde{f}_4(s_4)$	s_3^*
⑧	$(74, 76, 80, 81, 84, 91)$	$(111, 122, 134, 142, 150, 165)$	$(47, 53, 60, 69, 78, 84)$	$(47, 53, 60, 69, 78, 84)$	⑦

Thus, we obtain the optimal value of $\tilde{f}4(s4)$ in stage 4, which is $(47, 53, 60, 69, 78, 84)$, associated with state $s_4 =$ ⑧. This set of values represents the minimal distances for reaching state ⑧ and reflects the best routes available at this stage.
Notably, the results from dynamic programming match those from the ranking function, confirming the reliability of our method. This consistency shows how well our approach handles uncertainties and improves transportation logistics.

6. Comparison with existing methods

Table 7 presents a comprehensive comparison between the solutions obtained using the ranking function employed in this study and those derived from various existing methods found in the literature. The comparison highlights key performance indicators, including the least-cost path, the associated costs, the capacity to manage uncertainty in activity durations, and the overall computational efficiency in determining the least-cost path. This comparative analysis demonstrates that the proposed method consistently yields more cost-efficient solutions, establishing it as a competitive alternative to the other approaches considered.

1. Deng et al. [34] utilized the graded mean integration method. Following their approach, we apply graded mean integration to a hexagonal fuzzy number $\tilde{I}_h = (i_1, i_2, i_3, i_4, i_5, i_6; w)$. The graded mean integration in this case is expressed as follows:

$$GM = \frac{w(i_1 + 2i_2 + 3i_3 + 3i_4 + 2i_5 + i_6)}{12}$$

2. Nagalakshmi et al. [4] proposed an approach to solve the Generalized Trapezoidal Fuzzy Least-Cost Route problem using the centroid method. In this paper, we extend their method by applying the centroid technique for Hexagonal Fuzzy Numbers, employing the corresponding formula to achieve optimal route selection.

$$C(\tilde{I}_h) = \frac{w(i_1 + i_2 + i_3 + i_4 + i_5 + i_6)}{6}$$

3. Ramkumar et al. [35] adopt a statistical approach to the hexagonal fuzzy shortest travelling path problem, utilizing measures of central tendency. This method closely aligns with the approach employed by [4], indicating a similarity in their techniques for addressing fuzzy shortest path problems.

4. Rajkumar et al. [36] determined the shortest path in an acyclic network using hexagonal, heptagonal, and octagonal fuzzy numbers. The results were compared by defuzzifying the fuzzy numbers through the magnitude measure, providing an effective approach for handling complex fuzzy representations in network optimization problems.

$$GM = \frac{w(-i_1 + 6i_2 + i_3 + i_4 + 6i_5 - i_6)}{12}$$

The methodologies reviewed in this paper present a comprehensive framework for addressing fuzzy shortest path and least-cost route problems utilizing various types of fuzzy numbers. Specifically, the extension of the centroid method to hexagonal fuzzy numbers facilitates an effective mechanism for optimal route determination, enhancing the existing literature on fuzzy optimization. The application of graded mean integration, the centroid technique, and magnitude measures exemplifies the versatility of these approaches in managing uncertainty inherent in network optimization scenarios.

Furthermore, the close alignment between the techniques proposed by Deng et al. [34], Nagalakshmi et al. [4], Ramkumar et al. [35], and Rajkumar et al. [36] underscores their collective effectiveness in tackling complex fuzzy shortest path problems. Each method contributes uniquely to the field, offering insights into the performance of different fuzzy representations in route selection. This comparative analysis not only highlights the strengths and limitations of the various approaches but also serves as a foundation for future research aimed at refining these methodologies. Consequently, this paper not only enriches the discourse on fuzzy optimization but also paves the way for further advancements in the study of fuzzy shortest path problems.

Table 7: *Comparison table*

Methods	Least Cost Path	Least Cost
Deng et al. [34]	P_3 : ①→④→⑦→⑧	65.00
Nagalakshmi et al. [4]	P_3 : ①→④→⑦→⑧	65.16
Ramkumar et al. [35]	P_3 : ①→④→⑦→⑧	65.17
Rajkumar et al. [36]	P_1 : ①→③→⑥→⑧	65.33
Proposed Method	P_3 : ①→④→⑦→⑧	18.07

The comparison of the least cost values for the generalized hexagonal fuzzy number, derived from our proposed approach and those from [36], is presented in Table 7. The table demonstrates that our results consistently exceed those obtained by [36], underscoring the advantages inherent in our method. This superior performance can be attributed to the innovative aspects of our approach, which distinctly differentiate it from the methodology employed by [36].

To further substantiate this claim, we have implemented the conventional methodology detailed in Section 4.2. The results acquired through this traditional method provide an additional layer of verification, reinforcing the accuracy and reliability of our proposed solution in comparison to existing approaches. This comprehensive evaluation not only highlights the efficacy of our proposed method but also emphasizes its potential for advancing the field of fuzzy optimization in route selection problems. The findings suggest that our approach offers a more effective alternative for practitioners and researchers seeking to optimize least-cost routes using generalized hexagonal fuzzy numbers.

7. Conclusion and Future Research

Fuzzy ranking techniques demonstrate a superior capacity for managing uncertainties and imprecise data compared to deterministic methods, thereby enhancing the robustness of solutions against variations and errors in input parameters. This study presents a numerical case in which cost values are represented by generalized hexagonal fuzzy numbers. We derive these values in both crisp and fuzzy forms utilizing the ranking technique and fuzzy dynamic programming. The adaptability of the ranking function allows for updates or modifications in response to changing conditions or priorities, facilitating dynamic optimization in practical applications.

Our approach outperforms the results of the existing study by [36], showcasing its effectiveness. Additionally, we employ the dynamic programming method as a means of validation, demonstrating the efficacy of our proposed solution. By ranking perceptions according to the relative significance of various factors, our method yields more comprehensible outcomes and promotes transparent decision-making processes.

Future research endeavors may build upon this study by exploring the representation of activity durations using alternative fuzzy number types, further enhancing the applicability and versatility of fuzzy ranking techniques in optimization problems.

Compliance with Ethical Standards:

Declarations

Funding The authors did not receive support from any organization for the submitted work.

Conflict of interest All Authors declare that they have no conflict of interest.

Research involving human participants and/or animals The authors declare that this project does not involve Human Participants and/or animals in any capacity.

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