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Уточненная квазитрехмерная теория функционально-градиентных пористых пластин: смещения и напряжения

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В работе представлена теория сдвиговой и нормальной деформации высокого порядка для решения статической задачи функционально-градиентных пористых толстых прямоугольных пластин. Учитывается влияние растяжения пористых пластин в продольном направлении. Свойства функционально-градиентного пористого материала варьируются по толщине пластин. С использованием принципа виртуальных перемещений получены определяющие уравнения. Статическая задача решается для свободно опертой пластины при синусоидальной нагрузке. Получены точные выражения для смещений и напряжений. Обсуждается влияние градиентного параметра в экспоненциальной форме и коэффициента пористости на смещения и напряжения пористых пластин. Даны примеры, обосновывающие точность данной 3D теории в предсказании отклика на изгиб пористых пластин. Эффективность представленной модели оценивается на основе численных результатов для смещений и напряжений функционально-градиентных пористых пластин. Полевые переменные функционально-градиентных пластин очень чувствительны к изменению коэффициента пористости.

Ключевые слова: пористые пластины, функционально-градиентный материал, уточненная теория, смещения, напряжения

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Quasi-3D refined theory for functionally graded porous plates: Displacements and stresses

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This paper presents a higher-order shear and normal deformation theory for the static problem of functionally graded porous thick rectangular plates. The effect of thickness stretching in the functionally graded porous plates is taken into consideration. The functionally graded porous material properties vary through the plate thickness with a specific function. The governing equations are obtained via the virtual displacement principle. The static problem is solved for a simply supported plate under a sinusoidal load. The exact expressions for displacements and stresses are obtained. The influences of the functionally graded and porosity factors on the displacements and stresses of porous plates are discussed. Some validation examples are presented to show the accuracy of the present quasi-3D theory in predicting the bending response of porous plates. The effectiveness of the present model is evaluated by numerical results that include displacements and stresses of functionally graded porous plates. The field variables of functionally graded plates are very sensitive to the variation of the porosity factor.

Keywords: porous plates, functionally graded materials, refined theory, displacements, stresses

1. Introduction

Porous material beams, plates, and/or shells are widely used in structural design problems. The permeability, tensile strength, and electrical conductivity depend on the properties of the matrix and fluid in pores, in so doing porosity is regarded as the main property of the porous material. Numerous theoretical and experimental studies have been carried out on porous structures, which operate as novel multifunctional ultralight structural forms with new mechanical properties.

Biot was the first who developed a 3D model of wave propagation in fluid-saturated porous elastic structures [1]. In the meantime, poroelasticity has been successfully used in studies of various problems in engineering structures [2, 3]. Taber performed the quasi-static analysis of a poroelastic plate by applying the classical Kirchhoff plate theory [4]. As it is well known, Kirchhoff's assumptions are invalid for thick plates. Therefore, Busse et al. used the first-order Mindlin plate theory to model moderately thick plates based on Biot's poroelastic theory without presenting numerical

results [5]. Borsan used the porous plate theory to propose field equations for bending of thermoelastic plates [6]. Magnucki and Stasiewicz investigated elastic bending of isotropic porous beams with nonlinear hypotheses [7]. Nappa and Iesan presented thermoelastic problems of porous circular pipes and cylinders [8]. Magnucki et al. discussed bending and buckling of a rectangular porous plate taking into account the effect of shear deformation [9]. Magnucka-Blandzi discussed the nonlinear dynamic stability, deflection, and buckling of porous plates [10]. Yang et al. discussed quasi-static and dynamic bending of porous beams under step loads at their free ends [11]. Borsan and Altenbach presented the theory of porous elastic and thermoelastic thin rods modeled by the direct approach [12, 13]. Kumar and Devi presented transient problems for thermoelastic porous materials taking into account the temperature dependence of material properties [14]. Ghiba considered bending of thermoelastic porous plates of the Mindlin type [15]. Sladek et al. presented the first-order Mindlin theory for the bending response of porous plates according to Biot's poroelastic theory [16]. Lyapin and Vatulyan developed a mathematical model to study deformation of porous plates [17].

In recent years, functionally graded materials (FGMs) have found other applications in electrical appliances, energy transformation, biomedical engineering, optics, etc. However, in the manufacture of FGMs, porosities may occur in the materials during the sintering process. Chen et al. performed the buckling and bending analyses of functionally graded porous beams using the first-order beam theory [18]. Bensaïd and Guenanou [19] applied the nonlocal Timoshenko beam model to present deflection and buckling of functionally graded nanoscale beams with porosity. Akbas dealt with nonlinear static deflections of functionally graded porous beams under the thermal effect with position- and temperature-dependent material properties [20]. Behravan Rad presented the static response of porous multidirectional heterogeneous structures resting on developed gradient elastic foundations [21]. Barati et al. [22] and Barati and Zenkour [23] discussed the electromechanical vibration of smart piezoelectric functionally graded plates with porosities. Barati and Zenkour presented the

postbuckling behavior of beams reinforced with graphene platelets with porosities and geometrical imperfection [24]. Barati and Zenkour discussed the nonlocal strain gradient elasticity for wave propagation in graded nanoporous double nanobeam systems on the elastic foundation [25, 26].

The present paper obtains displacements and stresses in functionally graded porous plates subjected to a sinusoidal distributed load. It is assumed that the material properties of the porous plate are changed through the plate thickness. The plate is graded according to a novel type of the polynomial law. The plate faces are perfectly homogeneous while the whole plate has a perfect porous homogeneous shape according to the volume fraction of voids (porosity) or the graded factors. The variational principle is used to derive the governing equations based on the quasi-3D theory. Several important aspects that affect displacements and stresses are discussed in details.

2. Formulation of the problem

2.1. Structural model

Consider an functionally graded thick rectangular plate with thickness h , length a , and width b , as depicted in Fig. 1. The Cartesian coordinate system is established so that $0 \leq x \leq a$, $0 \leq y \leq b$, and $-h/2 \leq z \leq h/2$. Let the plate be under a distributed load $q(x, y)$ at the upper face $z = h/2$.

Young's modulus $E(z)$ may vary continuously through the plate thickness by means of the polynomial material law [27]:

$$E = E_c, \quad p = 0,$$

$$E = E_m \left[1 - \left(\frac{z}{h} + \frac{1}{2} \right)^p - \frac{\alpha}{2} \right] + E_c \left[\left(\frac{z}{h} + \frac{1}{2} \right)^p - \frac{\alpha}{2} \right], \quad (1)$$

$$p > 0, \quad 0 \leq \alpha \ll 1,$$

$$E = E_m, \quad p \rightarrow \infty,$$

where α represents the porosity volume function and p is the scalar parameter that defines gradation of material properties in the through-thickness direction, E_m and E_c are Young's moduli of the lower (as metal) and upper (as ceramic) faces of a functionally graded plate, respectively. In the case of $p = 0$ or $p \rightarrow \infty$, the plate becomes purely homogeneous (ceramic or metal) without any porosity.

Fig. 1. Geometry and coordinates of the functionally graded porous plate

2.2. Quasi-3D theory

The quasi-3D shear deformation theory for plates is suitable for the following displacements [27–32]:

$$\begin{aligned} u(x, y, z) &= u_0(x, y) - z \frac{\partial w}{\partial x} + \psi(z)u_1(x, y), \\ v(x, y, z) &= v_0(x, y) - z \frac{\partial w}{\partial y} + \psi(z)v_1(x, y), \\ w(x, y, z) &= w_0(x, y) + \psi'(z)w_1(x, y), \end{aligned} \quad (2)$$

where (u, v, w) denote the displacement field of a general point (x, y, z) in the functionally graded plate, (u_0, v_0, w_0) denote the displacement projections on the midplane, u_1 and v_1 are the rotations about the y and x axes, and w_1 represents the additional unknown variable. The variables u_0, v_0, w_0, u_1, v_1 , and w_1 are the functions of x and y . The prime ' represents differentiation with respect to z . The present theory considers a new perception of $\psi(z)$ as

$$\psi(z) = \frac{zh^2}{5h^2 + 4z^2} - \frac{4z^3}{27h^2}, \quad (3)$$

which comprises only one additional dependent unknown ($w_1 \neq 0$) in contrast to other theories but takes account of the effect of transverse normal strain through the plate thickness. No shear correction factors are needed for the present theory. According to the displacements in Eq. (2), the linear strains ϵ_i ($\epsilon_1 = \epsilon_{11}, \epsilon_2 = \epsilon_{22}, \epsilon_3 = \epsilon_{33}, \epsilon_4 = \epsilon_{23}, \epsilon_5 = \epsilon_{13}, \epsilon_6 = \epsilon_{12}$) are presented as

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{Bmatrix} = \begin{Bmatrix} \epsilon_1^{(0)} \\ \epsilon_2^{(0)} \\ \epsilon_6^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \epsilon_1^{(1)} \\ \epsilon_2^{(1)} \\ \epsilon_6^{(1)} \end{Bmatrix} + \psi(z) \begin{Bmatrix} \epsilon_1^{(2)} \\ \epsilon_2^{(2)} \\ \epsilon_6^{(2)} \end{Bmatrix}, \quad (4)$$

$$\{\epsilon_4, \epsilon_5\} = \psi'(z) \{\epsilon_4^{(0)}, \epsilon_5^{(0)}\}, \quad \epsilon_3 = \psi''(z) \epsilon_3^{(0)}.$$

Here $\epsilon_i^{(0)}, \epsilon_i^{(1)}$, and $\epsilon_i^{(2)}$ are expressed as

$$\begin{aligned} \epsilon_1^{(0)} &= \frac{\partial u_0}{\partial x}, \quad \epsilon_2^{(0)} = \frac{\partial v_0}{\partial y}, \quad \epsilon_3^{(0)} = w_1, \quad \epsilon_4^{(0)} = v_1 + \frac{\partial w_1}{\partial y}, \\ \epsilon_5^{(0)} &= u_1 + \frac{\partial w_1}{\partial x}, \quad \epsilon_6^{(0)} = \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y}, \quad \epsilon_1^{(1)} = -\frac{\partial^2 w_0}{\partial x^2}, \\ \epsilon_2^{(1)} &= -\frac{\partial^2 w_0}{\partial y^2}, \quad \epsilon_6^{(1)} = -2\frac{\partial^2 w_0}{\partial x \partial y}, \quad \epsilon_1^{(2)} = \frac{\partial u_1}{\partial x}, \\ \epsilon_2^{(2)} &= \frac{\partial v_1}{\partial y}, \quad \epsilon_6^{(2)} = \frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y}. \end{aligned} \quad (5)$$

The stress-strain constitutive equations for the present porous plate are represented as

$$\sigma_i = \lambda(z)[\epsilon_1 + \epsilon_2 + \epsilon_3]\delta_i + 2\mu(z)\epsilon_i, \quad (6)$$

where $\delta_i = 1$ for $i = 1, 2, 3$, $\delta_i = 0$ for $i = 4, 5, 6$, $\lambda(z)$ and $\mu(z)$ are the Lamé coefficients given by

$$\lambda(z) = \frac{\nu E(z)}{(1+\nu)(1-2\nu)}, \quad \mu(z) = \frac{E(z)}{2(1+\nu)}, \quad (7)$$

and $E(z)$ varies in the through-thickness direction of the porous plate as discussed previously in Eq. (1) while ν represents a fixed value of the Poisson ratio.

3. Equilibrium equations

The static equations can be obtained by applying the principle of virtual displacements. It can be stated in its analytical form as

$$\begin{aligned} &\iint_{\Omega} \left\{ \int_{-h/2}^{h/2} \sigma_i \delta \epsilon_i dz - q[\delta w_0 + \psi'(z)\delta w_1]_{z=-h/2}^{z=h/2} \right\} d\Omega = 0 \quad (8) \\ \text{or} &\iint_{\Omega} \left[N_1 \frac{\partial \delta u_0}{\partial x} + N_2 \frac{\partial \delta v_0}{\partial y} + N_6 \left(\frac{\partial \delta u_0}{\partial y} + \frac{\partial \delta v_0}{\partial x} \right) + \right. \\ &+ N_3 \delta w_1 - M_1 \frac{\partial^2 \delta w_0}{\partial x^2} - M_2 \frac{\partial^2 \delta w_0}{\partial y^2} - \\ &- 2M_6 \frac{\partial^2 \delta w_0}{\partial x \partial y} + S_1 \frac{\partial \delta u_1}{\partial x} + S_2 \frac{\partial \delta v_1}{\partial y} + \\ &+ S_6 \left(\frac{\partial \delta v_1}{\partial x} + \frac{\partial \delta u_1}{\partial y} \right) + Q_5 \left(\delta u_1 + \frac{\partial \delta w_1}{\partial x} \right) + \\ &+ Q_4 \left(\delta v_1 + \frac{\partial \delta w_1}{\partial y} \right) - q \delta w_0 \Big] d\Omega = 0, \quad (9) \end{aligned}$$

where N_j and M_j ($j = 1, 2, 6$) are the stress resultants and stress couples, S_j is the additional stress couples, and Q_l ($l = 4, 5$) and N_3 are the transverse and normal shear stress resultants. They are introduced in Eq. (6) as

$$\{N_j, M_j, S_j\} = \int_{-h/2}^{h/2} \{1, z, \psi(z)\} \sigma_j dz, \quad j = 1, 2, 6, \quad (10)$$

$$Q_l = \int_{-h/2}^{h/2} \psi'(z) \sigma_l dz, \quad N_3 = \int_{-h/2}^{h/2} \psi''(z) \sigma_3 dz.$$

Then, by integrating Eq. (9) by parts and putting the coefficients of $\delta u_0, \delta v_0, \delta w_0, \delta u_1, \delta v_1$, and δw_1 to zero, separately. Thus, according to the present theory we have

$$\begin{aligned} \delta u_0: \quad &\frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0, \quad \delta v_0: \quad \frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} = 0, \\ \delta w_0: \quad &\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_6}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} + q = 0, \\ \delta u_1: \quad &\frac{\partial S_1}{\partial x} + \frac{\partial S_6}{\partial y} - Q_5 = 0, \\ \delta v_1: \quad &\frac{\partial S_6}{\partial x} + \frac{\partial S_2}{\partial y} - Q_4 = 0, \\ \delta w_1: \quad &\frac{\partial Q_5}{\partial x} + \frac{\partial Q_4}{\partial y} - N_3 = 0. \end{aligned} \quad (11)$$

In addition, the natural boundary conditions may be represented by

$$\begin{aligned} \delta u_0: \quad &N_1 n_x + N_6 n_y, \quad \delta v_0: \quad N_6 n_x + N_2 n_y, \\ \delta u_1: \quad &S_1 n_x + S_6 n_y, \quad \delta v_1: \quad S_6 n_x + S_2 n_y, \\ \delta w_0: \quad &\left(\frac{\partial M_{11}}{\partial x} + \frac{\partial M_6}{\partial y} \right) n_x + \\ &+ \left(\frac{\partial M_6}{\partial x} + \frac{\partial M_2}{\partial y} \right) n_y + \frac{\partial M_{ns}}{\partial s}, \end{aligned} \quad (12)$$

$\delta w_1: Q_5 n_x + Q_4 n_y, \frac{\partial \delta w_0}{\partial n}: M_n,$
where

$$\begin{aligned} M_{ns} &= (M_2 - M_1) n_x n_y + M_6 (n_x^2 - n_y^2), \\ M_n &= M_1 n_x^2 + M_2 n_y^2 + 2 M_6 n_x n_y, \\ \frac{\partial}{\partial n} &= n_x \frac{\partial}{\partial x} + n_y \frac{\partial}{\partial y}. \end{aligned} \quad (13)$$

The stress resultants can be obtained in terms of the total strains by substituting Eq. (6) into Eq. (10):

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \\ S_1 \\ S_2 \\ S_6 \\ N_3 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 & G_{13}^s \\ A_{12} & A_{11} & 0 & B_{12} & B_{11} & 0 & B_{12}^s & B_{11}^s & 0 & G_{13}^s \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s & 0 \\ B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{13}^s \\ B_{12} & B_{11} & 0 & B_{12}^s & B_{11}^s & 0 & D_{12}^s & D_{11}^s & 0 & H_{13}^s \\ 0 & 0 & B_{66} & 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & F_{11}^s & F_{12}^s & 0 & J_{13}^s \\ B_{12}^s & B_{11}^s & 0 & D_{12}^s & D_{11}^s & 0 & F_{12}^s & F_{11}^s & 0 & J_{13}^s \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & F_{66}^s & 0 \\ G_{13}^s & G_{13}^s & 0 & H_{13}^s & H_{13}^s & 0 & J_{13}^s & J_{13}^s & 0 & P_{33}^s \end{bmatrix} \begin{Bmatrix} \partial u_0 / \partial x \\ \partial v_0 / \partial y \\ \partial v_0 / \partial x + \partial u_0 / \partial y \\ -\partial^2 w_0 / \partial x^2 \\ -\partial^2 w_0 / \partial y^2 \\ -2\partial^2 w_0 / \partial x \partial y \\ \partial u_1 / \partial x \\ \partial v_1 / \partial y \\ \partial v_1 / \partial x + \partial u_1 / \partial y \\ w_1 \end{Bmatrix} \quad (14)$$

$$\{Q_4, Q_5\} = A_{44}^s \left\{ \psi_2 + \frac{\partial \psi_3}{\partial y}, \psi_1 + \frac{\partial \psi_3}{\partial x} \right\}, \quad (15)$$

where

$$\begin{aligned} \{A_{11}, B_{11}, B_{11}^s, D_{11}, D_{11}^s, F_{11}^s\} &= \int_{-h/2}^{h/2} [\lambda(z) + 2\mu(z)] \{1, z, \psi(z), z^2, z\psi(z), [\psi(z)]^2\} dz, \\ \{A_{12}, B_{12}, B_{12}^s, D_{12}, D_{12}^s, F_{12}^s\} &= \int_{-h/2}^{h/2} \lambda(z) \{1, z, \psi(z), z^2, z\psi(z), [\psi(z)]^2\} dz, \\ \{A_{66}, B_{66}, B_{66}^s, D_{66}, D_{66}^s, F_{66}^s\} &= \int_{-h/2}^{h/2} \mu(z) \{1, z, \psi(z), z^2, z\psi(z), [\psi(z)]^2\} dz, \\ A_{44}^s &= \int_{-h/2}^{h/2} \mu(z) [\psi'(z)]^2 dz, \\ P_{33}^s &= \int_{-h/2}^{h/2} [\lambda(z) + 2\mu(z)] [\psi''(z)]^2 dz, \\ \{G_{13}^s, H_{13}^s, J_{13}^s\} &= \int_{-h/2}^{h/2} \lambda(z) \{1, z, \psi(z)\} \psi''(z) dz. \end{aligned} \quad (16)$$

Substituting Eqs. (14) and (15) into Eq. (11) yields a system of algebraic equations expressed in a compact form as

$$[k] \{\Delta\} = \{f\}, \quad (17)$$

where $[k]$ represents the symmetric matrix of differential operators and $\{f\} = \{0, 0, q, 0, 0, 0\}^t$ denotes the generalized force vector while $\{\Delta\} = \{u_0, v_0, w_0, u_1, v_1, w_1\}^t$. The elements $k_{ij} = k_{ji}$ of the symmetric matrix $[k]$ are written in Appendix A.

4. Two-dimensional solution

The following simply supported boundary conditions are set at the side edges of the functionally graded plate:

$$\begin{aligned} v_0 = w_0 = v_1 = w_1 = N_1 = M_1 = S_1 = 0 & \text{ at } x = 0, a, \\ u_0 = w_0 = u_1 = w_1 = N_2 = M_2 = S_2 = 0 & \text{ at } y = 0, b. \end{aligned} \quad (18)$$

The external force according to the Navier solution can be expressed in the sinusoidal form as

$$q(x, y) = q_0 \sin(\xi x) \sin(\eta y), \quad (19)$$

where $\xi = \pi/a$, $\eta = \pi/b$, and q_0 denotes the load intensity in the plate centre. According to the Navier technique, one assumes the following forms for u_0, v_0, w_0, u_1, v_1 , and w_1 that satisfy the above conditions:

$$\begin{Bmatrix} (u_0, u_1) \\ (w_0, w_1) \\ (v_0, v_1) \end{Bmatrix} = \begin{Bmatrix} (U, X) \cos(\xi x) \sin(\eta y) \\ (W, Z) \sin(\xi x) \sin(\eta y) \\ (V, Y) \sin(\xi x) \cos(\eta y) \end{Bmatrix}, \quad (20)$$

where U, V, W, X, Y , and Z are the arbitrary parameters. Equation (11) can be combined with Eqs. (14) and (15) into the system of first-order equations

$$[K] \{\bar{\Delta}\} = \{F\}, \quad (21)$$

where $\{\bar{\Delta}\}$ and $\{F\}$ denote the columns

$$\{\bar{\Delta}\} = \{U, V, W, X, Y, Z\}^t, \quad \{F\} = \{0, 0, q_0, 0, 0, 0\}^t, \quad (22)$$

and the elements $K_{ij} = K_{ji}$ of the coefficient matrix $[K]$ are written in Appendix B.

5. Numerical results and discussion

The bending response of simply-supported functionally graded porous thick rectangular plates subjected to a sinu-

soidally distributed load is investigated. The effect of thickness stretching ε_3 of the functionally graded porous plates is taken into account in the present quasi-3D theory. The displacements and stresses determined here are reported according to the following dimensionless forms:

$$\begin{aligned}\bar{w}(\bar{z}) &= \frac{10E_c h^3}{a^4 q_0} w\left(\frac{a}{2}, \frac{b}{2}, z\right), \\ \bar{u}(\bar{z}) &= \frac{100E_c h^3}{a^4 q_0} u\left(\frac{a}{2}, \frac{b}{2}, z\right), \\ \bar{\sigma}_4(\bar{z}) &= \frac{h}{aq_0} \sigma_4\left(\frac{a}{2}, 0, z\right),\end{aligned}\quad (23)$$

$$\bar{\sigma}_5(\bar{z}) = \frac{h}{aq_0} \sigma_5\left(0, \frac{b}{2}, z\right), \quad \bar{z} = \frac{z}{h},$$

$$\bar{\sigma}_6(\bar{z}) = \frac{h}{aq_0} \sigma_6(0, 0, z),$$

$$\bar{\sigma}_i(\bar{z}) = \frac{h}{aq_0} \sigma_i\left(\frac{a}{2}, \frac{b}{2}, z\right), \quad i = 1, 2, 3.$$

The lower surface of the functionally graded plate is metal ($E_m = 70$ GPa) while the upper surface is ceramic ($E_c = 380$ GPa). Poisson's ratio is fixed at $\nu = 0.3$. As a validation example, the present deflection and in-plane

Table 1

Comparisons of deflection $\bar{w}(0)$ of functionally graded porous square plates

p	Methods		a/h		
			4	10	100
0	Ref. [36]		0.366500	0.294200	0.280300
	Present	$\alpha = 0.0$	0.366044	0.293938	0.280089
		$\alpha = 0.1$	0.389082	0.312438	0.297717
0.5	Ref. [36]		0.549300	0.454800	0.436500
	Present	$\alpha = 0.0$	0.541750	0.445172	0.426618
		$\alpha = 0.1$	0.596922	0.491987	0.471828
1	CPT [36]		0.562300	0.562300	0.562300
	FSDT [36]		0.729100	0.588900	0.562500
	Ref. [33]		0.717100	0.587500	0.562500
	Ref. [34]		0.717100	0.587500	0.562500
	Ref. [35]		0.699700	0.584500	0.562400
	Ref. [36]		0.702000	0.586800	0.564700
	Present	$\alpha = 0.0$	0.690817	0.569038	0.545649
		$\alpha = 0.1$	0.791092	0.655782	0.629794
	Ref. [36]		0.702000	0.586800	0.564700
4	CPT [36]		0.828100	0.828100	0.828100
	FSDT [36]		1.112500	0.873600	0.828100
	Ref. [33]		1.158500	0.882100	0.828600
	Ref. [34]		1.158500	0.882100	0.828600
	Ref. [35]		1.117800	0.875000	0.828600
	Ref. [36]		1.110800	0.870000	0.824000
	Present	$\alpha = 0.0$	1.098263	0.841682	0.792489
		$\alpha = 0.1$	1.405700	1.082177	1.020157
	Ref. [36]		1.110800	0.870000	0.824000
10	CPT [36]		0.935400	0.935400	0.935400
	FSDT [36]		1.317800	0.996600	0.936000
	Ref. [33]		1.374500	1.007200	0.936100
	Ref. [34]		1.374500	1.007200	0.936100
	Ref. [35]		1.349000	0.875000	0.828600
	Ref. [36]		1.333400	0.988800	0.922700
	Present	$\alpha = 0.0$	1.335130	0.981809	0.914068
		$\alpha = 0.1$	1.784337	1.284910	1.189183
	Ref. [36]		1.333400	0.988800	0.922700

Table 2

Comparisons of in-plane normal stress $\bar{\sigma}_1(1/3)$ of functionally graded porous square plates

p	Methods		a/h		
			4	10	100
0	Ref. [36]		0.527800	1.317600	13.161000
	Present	$\alpha = 0.0$	0.522946	1.303085	13.019919
		$\alpha = 0.1$	0.522946	1.303085	13.019919
0.5	Ref. [36]		0.586000	1.468000	14.673000
	Present	$\alpha = 0.0$	0.571960	1.419381	14.170342
		$\alpha = 0.1$	0.577905	1.433585	14.311072
1	CPT [36]		0.806000	2.015000	20.150000
	FSDT [36]		0.806000	2.015000	20.150000
	Ref. [33]		0.622100	1.506400	14.969000
	Ref. [34]		0.622100	1.506400	14.969000
	Ref. [35]		0.592500	1.494500	14.969000
	Ref. [36]		0.591100	1.491700	14.945000
	Present	$\alpha = 0.0$	0.576180	1.434203	14.326085
		$\alpha = 0.1$	0.586141	1.458887	14.572361
4	CPT [36]		0.642000	1.604900	16.049000
	FSDT [36]		0.642000	1.604900	16.049000
	Ref. [33]		0.487700	1.197100	11.923000
	Ref. [34]		0.487700	1.197100	11.923000
	Ref. [35]		0.440400	1.178300	11.932000
	Ref. [36]		0.433000	1.158800	11.737000
	Present	$\alpha = 0.0$	0.425573	1.103477	11.104869
		$\alpha = 0.1$	0.396489	1.043439	10.528310
10	CPT [36]		0.479600	1.199000	11.990000
	FSDT [36]		0.479600	1.199000	11.990000
	Ref. [33]		0.369500	0.896500	8.907700
	Ref. [34]		0.347800	0.896500	8.907700
	Ref. [35]		0.322700	1.178300	11.93200
	Ref. [36]		0.309700	0.846200	8.601000
	Present	$\alpha = 0.0$	0.310215	0.824260	8.330706
		$\alpha = 0.1$	0.259359	0.716567	7.290624

stress will be compared with the classical plate theory (CPT), the first-order shear deformation theory (FSDT), that uses a shear correction factor $K = 5/6$ and those from Carrera et al. [33, 34] and Neves et al. [35, 36], which account for $\varepsilon_3 \neq 0$ and use the Carrera unified formulation (CUF). Note that displacement fields of the classical plate theory and first-order shear deformation theory are given from Eq. (2), respectively, by setting $\psi(z) = 0$ and $\psi(z) = z$, $w_1 = 0$. No shear correction factor is needed for the mentioned theories, except for the first-order shear deformation theory.

Tables 1 and 2 contain dimensionless deflection and in-plane stress in nonporous ($\alpha = 0$) functionally graded square plates for different side-to-thickness ratio a/h and graded parameter p . The inclusion of the porosity factor α is represented in these tables. The present nonporous results ($\alpha = 0$) are more accurate than those generated by the classical plate theory and first-order shear deformation theory. The present results are comparable with those of other quasi-3D theories even for thicker plates. This points to the use of the new assumption given in Eq. (3), which has a maximal effect on the accuracy of the results. Table 1 shows

Table 3

Comparisons of dimensionless stresses and displacements of functionally graded porous square plates ($a/h = 10$)

p	Methods		$\bar{u}(-1/4)$	$\bar{w}(0)$	$\bar{\sigma}_1(1/3)$	$\bar{\sigma}_3(1/6)$	$\bar{\sigma}_6(-1/3)$
0	Present	$\alpha = 0.0$	0.216337	0.293938	1.303085	0.212331	0.700122
		$\alpha = 0.1$	0.229953	0.312438	1.303085	0.212331	0.700121
1	ITST [41]		0.641300	0.589000	1.489000	0.261100	0.611100
	Quasi-3D [33]		0.643600	0.587500	1.506200	0.251000	0.608100
	Quasi-3D [38]		0.643600	0.587600	1.506100	0.251100	0.611200
	SSDT [37]		0.662600	0.588900	1.489400	0.262200	0.611000
	HSDT [39]		0.639800	0.588000	1.488800	0.256600	0.610900
	TSDT [40]		0.641400	0.589000	1.489800	0.260800	0.611100
	Present	$\alpha = 0.0$	0.587959	0.569038	1.434203	0.261088	0.565518
		$\alpha = 0.1$	0.701623	0.655782	1.458887	0.266506	0.546517
2	ITST [41]		0.898200	0.757300	1.395900	0.274200	0.544200
	Quasi-3D [33]		0.901200	0.757000	1.414700	0.249600	0.542100
	Quasi-3D [38]		0.901300	0.757100	1.413300	0.249500	0.543600
	SSDT [37]		0.928100	0.757300	1.395400	0.276300	0.544100
	HSDT [39]		0.895700	0.756400	1.394000	0.274100	0.543800
	TSDT [40]		0.898400	0.757300	1.396000	0.273700	0.544200
	Present	$\alpha = 0.0$	0.807013	0.721361	1.325404	0.274742	0.493683
		$\alpha = 0.1$	1.049624	0.885498	1.332625	0.284519	0.453813
4	ITST [41]		1.050000	0.881600	1.179200	0.254600	0.566900
	Quasi-3D [33]		1.054100	0.882300	1.198500	0.236200	0.566600
	Quasi-3D [38]		1.054100	0.882300	1.184100	0.236200	0.567100
	SSDT [37]		1.094100	0.881900	1.178300	0.258000	0.566700
	HSDT [39]		1.045700	0.881400	1.175500	0.262300	0.566200
	TSDT [40]		1.050200	0.881500	1.179400	0.253700	0.566900
	Present	$\alpha = 0.0$	0.942715	0.841682	1.103477	0.255831	0.514100
		$\alpha = 0.1$	1.310574	1.082177	1.043439	0.267216	0.479271
8	ITST [41]		1.075900	0.974600	0.947300	0.209400	0.585700
	Quasi-3D [33]		1.083000	0.973800	0.968700	0.226200	0.587900
	Quasi-3D [38]		1.083000	0.973900	0.962200	0.226100	0.588300
	SSDT [37]		1.134000	0.975000	0.946600	0.212100	0.585600
	HSDT [39]		1.070900	0.973700	0.943100	0.214000	0.585000
	TSDT [40]		1.076300	0.974600	0.947700	0.208800	0.585800
	Present	$\alpha = 0.0$	0.982899	0.944138	0.886117	0.210366	0.540509
		$\alpha = 0.1$	1.393026	1.232257	0.785381	0.210846	0.511665

that $\bar{w}$ decreases as a/h increases and the graded parameter p decreases. Table 2 shows that $\bar{\sigma}_1$ increases as a/h increases. Note that $\bar{\sigma}_1$ no longer increases as p increases and $\bar{\sigma}_1$ has its minimum values for homogeneous ceramic and metal materials.

In another validation example, Table 3 presents a comparison of the dimensionless displacements $\bar{u}$ and $\bar{w}$ and stresses $\bar{\sigma}_1$, $\bar{\sigma}_3$ and $\bar{\sigma}_6$ of nonporous ($\alpha = 0$) functionally

graded square plates ($a/h = 10$). The present displacements and stresses will be compared with the sinusoidal shear deformation theory (SSDT) [37], quasi-3D elasticity theory [28, 38], higher-order shear deformation theory (HSDT) [39], third-order shear deformation theory (TSDT) [40], and a new inverse trigonometric shear deformation theory (ITST) [41].

Note that displacements of the sinusoidal shear deformation theory [37] are given from Eq. (2) by putting

Fig. 2. Change of deflection $\bar{w}$ versus different parameters of functionally graded porous plates: $\bar{w}(z)$ across the plate thickness for different p (a) and α (b); $\bar{w}(0)$ versus a/h (c) and a/b (d) for different α .

$$\psi(z) = \frac{h}{\pi} \sin\left(\frac{\pi z}{h}\right) \text{ and } w_1 = 0. \quad (24)$$

Displacements of the higher-order shear deformation theory [39] are given by putting

$$\psi(z) = \frac{\pi}{2h} z + \sin\left(\frac{\pi z}{h}\right) e^{1/2 \cos(\pi z/h)}, \quad w_1 = 0. \quad (25)$$

Displacements of the third-order shear deformation theory [40] are given by putting

$$\psi(z) = z - \frac{4z^3}{3h^2}, \quad w_0 = w_b + w_s, \quad (26)$$

$$u_1 = \frac{\partial w_s}{\partial x}, \quad v_1 = \frac{\partial w_s}{\partial y}, \quad w_1 = 0.$$

Finally, displacements of the inverse trigonometric shear deformation theory [41] are given by putting

$$\psi(z) = h \arctg\left(\frac{rz}{h}\right) - \frac{16rz^3}{3h^2(r^2 + 4)}, \quad w_1 = 0, \quad (27)$$

where r is the arbitrary parameter.

The present results reported in Table 3 are in close agreement with various shear deformation theories. We can also say that the present theory is more accurate than those theories since it includes the effect of transverse normal strain ($\epsilon_3 \neq 0$). Table 3 shows that the displacements $\bar{u}$ and $\bar{w}$ increase as the graded parameter p increases. The in-plane normal stress $\bar{\sigma}_1$ and transverse shear stress $\bar{\sigma}_5$ no longer

increase as p increases. In addition, the tangential stress $\bar{\sigma}_6$ no longer decreases as p increases.

In fact, the present quasi-3D theory gives results comparable with the inclusion of the porosity factor ($\alpha = 0.1$). It is clear that this factor has a significant effect on displacements and stresses. The difference in deflections $\bar{w}$ obtained by the present quasi-3D theory with porosity ($\alpha = 0.1$) and nonporosity ($\alpha = 0$) may increase as p increases and a/h decreases. The porosity normal stress $\bar{\sigma}_1$ is greater than the nonporosity stress for functionally graded plates with $0 < p < 4$ and vice versa with $4 \leq p < \infty$. In addition, the porosity in-plane displacement $\bar{u}$ is greater than the nonporosity in-plane displacement $\bar{u}$ for functionally graded plates for all p . Finally, the porosity shear stress $\bar{\sigma}_5$ {tangential stress $\bar{\sigma}_6$ } is greater {smaller} than the nonporosity shear stress $\bar{\sigma}_5$ {tangential stress $\bar{\sigma}_6$ } for functionally graded plates for all p .

The inclusion of the porosity factor will be investigated in Figs. 2–9. In these figures we assume, unless otherwise stated, that $\alpha = 0.1$, $p = 5$, $a/h = 5$, and $b/a = 2$. All figures illustrate the variable quantities through the plate thickness versus the thickness ratio a/h and the aspect ratio a/b .

Figure 2, a shows that the deflection $\bar{w}$ increases as p decreases for the fixed $\alpha = 0.1$, while Fig. 2b shows that the deflection increases as α increases for the fixed $p = 0.1$. A porosity deflection value at $\alpha = 0.25$ may triple as com-

Fig. 3. Variation of in-plane displacement $\bar{u}$ versus different parameters of functionally graded porous plates: $\bar{u}(\bar{z})$ across the plate thickness for different p (a) and α (b); $\bar{u}(-1/4)$ versus a/h (c) and a/b (d) for different α

Fig. 4. Change of in-plane normal stress $\bar{\sigma}_1$ versus different parameters of functionally graded porous plates: $\bar{\sigma}_1(\bar{z})$ across the plate thickness for different p (a) and α (b); $\bar{\sigma}_1(1/2)$ versus a/h (c) and a/b (d) for different α

Fig. 5. Variation of the in-plane longitudinal stress $\bar{\sigma}_2$ versus different parameters of functionally graded porous plates: $\bar{\sigma}_2(\bar{z})$ across the plate thickness for different p (a) and α (b); $\bar{\sigma}_2(1/2)$ versus a/h (c) and a/b (d) for different α

Fig. 6. Variation of the out-of-plane normal stress $\bar{\sigma}_3$ versus different parameters of functionally graded porous plates: $\bar{\sigma}_3(\bar{z})$ across the plate thickness for different p (a) and α (b); $\bar{\sigma}_3(1/2)$ versus a/h (c) and a/b (d) for different α

Fig. 7. Change of transverse shear stress $\bar{\sigma}_4$ versus different parameters of functionally graded porous plates: $\bar{\sigma}_4(\bar{z})$ across the plate thickness for different p (a) and α (b); $\bar{\sigma}_4(0)$ versus a/h (c) and a/b (d) for different α

Fig. 8. Change of transverse shear stress $\bar{\sigma}_5$ versus different parameters of functionally graded porous plates: $\bar{\sigma}_5(\bar{z})$ across the plate thickness for different p (a) and α (b); $\bar{\sigma}_5(0)$ versus a/h (c) and a/b (d) for different α

Fig. 9. Variation of the in-plane tangential stress $\bar{\sigma}_6$ versus different parameters of functionally graded porous plates: $\bar{\sigma}_6(\bar{z})$ across the plate thickness for different p (a) and α (b); $\bar{\sigma}_6(-1/2)$ versus a/h (c) and a/b (d) for different α .

pared to nonporosity one. Figure 2, c shows that the deflection decreases slightly as a/h increases, while Fig. 2, d shows that the deflection decreases rapidly as a/b increases. The differences between deflections may be fixed at the variation of a/h while these differences are larger for small aspect ratios.

Figure 3, a shows that the porosity metallic in-plane displacement $\bar{u}$ at the lower surface $\bar{z} = -0.5$ is the greatest one while the porosity ceramic in-plane displacement $\bar{u}$ is the smallest one. The porosity FG in-plane displacement $\bar{u}$ increases as p increases. The inverse occurs at the upper surface $\bar{z} = 0.5$. The same occurs in Fig. 3, b. It is seen from Fig. 3, b that the variation of the porosity factor has no effect on the in-plane displacement $\bar{u}$ at the level of $\bar{z} = 1/3$. Figures 3, c and 3, d show that the in-plane displacement $\bar{u}$ decreases as a/h and a/b increase. The differences between the in-plane displacements $\bar{u}$ are larger for small thickness a/h and aspect a/b ratios. The variation of the porosity factor may have no effect on $\bar{u}$ for larger aspect ratios ($a/b > 3$).

Figure 4, a shows that the porosity metallic in-plane normal stress $\bar{\sigma}_1$ is the same as the porosity ceramic in-plane normal stress. Figure 4, b shows that the porosity functionally graded normal stress $\bar{\sigma}_1$ increases as α decreases in the interval $-0.16 < \bar{z} < 0.38$. Otherwise, $\bar{\sigma}_1$ increases as α increases. Figures 4, c and 4, d show that $\bar{\sigma}_1$ increases as

a/h increases and a/b decreases. The normal stress $\bar{\sigma}_1$ increases as the porosity factor α increases. The differences between the in-plane normal stresses $\bar{\sigma}_1$ are larger for a large thickness ratio a/h and a small aspect ratio a/b . Figure 5 shows that the in-plane longitudinal stress $\bar{\sigma}_2$ behaves similarly to the normal stress $\bar{\sigma}_1$. The maximum in-plane longitudinal stress $\bar{\sigma}_2$ occurs at $a/b = 0.6$ as shown in Fig. 5, d.

Once again Fig. 6, a shows that the porosity metallic out-of-plane transverse normal stress $\bar{\sigma}_3$ is the same as the porosity ceramic one. Figure 6, b shows that the porosity functionally graded transverse normal stress $\bar{\sigma}_3$ is very sensitive to the variation of the porosity factor α at the level $\bar{z} = 0.38$. Figure 6, c shows that $\bar{\sigma}_3$ increases as a/h increases for $\alpha = 0.25$. At $\alpha = 0.2$, $\bar{\sigma}_3$ no longer decreases as a/h increases and the minimum value of $\bar{\sigma}_3$ occurs at $a/h = 6.8$. However, for $\alpha \leq 0.15$ the out-of-plane transverse normal stress $\bar{\sigma}_3$ directly decreases as a/h increases. Figure 6, d shows that $\bar{\sigma}_3$ slightly increases as a/b increases only for nonporous plates ($\alpha = 0$). When $\alpha = 0.1$, $\bar{\sigma}_3$ is not affected by the variation of the aspect ratio a/b . However, at $\alpha \geq 0.15$ $\bar{\sigma}_3$ decreases as a/b increases.

Figures 7 and 8 show that the transverse shear stresses $\bar{\sigma}_4$ and $\bar{\sigma}_5$ have the same behavior through the thickness of the functionally graded porous and nonporous plates. The shear stresses are the same for ceramic and metallic

homogeneous plates. The maximum shear stresses have different positions according the value of the graded parameter p . However, the maximum shear stresses occur at the position ($\bar{z} = 1/3$) for different porosity factors α . In this position, the maximum shear stress increases as α increases. The transverse shear stresses are affected by the variation of a/h . Finally, $\bar{\sigma}_5$ directly decreases as a/b increases while $\bar{\sigma}_4$ no longer increases as a/b increases and has its maximum value at $a/b = 1$. In the meantime, the transverse shear stresses decrease as α increases.

Figure 9, *a* shows that the porosity metallic in-plane tangential stress $\bar{\sigma}_6$ is the same as the porosity ceramic one. Figure 9, *b* shows that the porosity functionally graded tangential stress $\bar{\sigma}_6$ increases as α increases in the interval $-0.16 < \bar{z} < 0.38$. Otherwise, $\bar{\sigma}_6$ increases as α decreases. Figure 4, *c* shows that $\bar{\sigma}_6$ increases as a/h increases. In addition, the tangential stress $\bar{\sigma}_1$ increases as the porosity factor α decreases. The differences between the in-plane tangential stresses $\bar{\sigma}_6$ are larger for a large thickness ratio a/h and small aspect ratio a/b . Once again, the maximum in-plane tangential stress $\bar{\sigma}_6$ occurs at $a/b = 0.6$, as shown in Fig. 9, *d*.

6. Conclusions

A refined quasi-3D shear and normal deformation theory is developed for functionally graded porous plates. The effect of thickness stretching in functionally graded porous plates is presented. The governing equations are derived and analytical solutions for simply supported functionally graded porous rectangular plates are obtained. The inclusion of the graded and porosity parameters is investigated. Many validation examples are reported, and numerical results of the present quasi-3D theory are accurate in predicting the bending response of nonporous plates. All stresses for porous or nonporous homogeneous plates are the same. That is because the stresses are independent of Young's modulus. The displacements and stresses are very sensitive to the variation of the porosity factor for functionally graded plates. It should be noted that the inclusion of the porosity factor is the main property of the porous materials, especially for functionally graded structures.

7. Appendices

7.1. Appendix A

The elements of the symmetric matrix $[k]$ presented in Eq. (17) are given by

$$\begin{aligned} k_{11} &= A_{11} \frac{\partial^2}{\partial x^2} + A_{66} \frac{\partial^2}{\partial y^2}, \quad k_{12} = (A_{11} + A_{66}) \frac{\partial^2}{\partial x \partial y}, \\ k_{13} &= -B_{11} \frac{\partial^3}{\partial x^3} - (B_{12} + 2B_{66}) \frac{\partial^3}{\partial x \partial y^2}, \\ k_{14} &= B_{11}^s \frac{\partial^2}{\partial x^2} + B_{66}^s \frac{\partial^2}{\partial y^2}, \quad k_{15} = (B_{12}^s + B_{66}^s) \frac{\partial^2}{\partial x \partial y}, \end{aligned}$$

$$\begin{aligned} k_{16} &= G_{13}^s \frac{\partial}{\partial x}, \quad k_{22} = A_{66} \frac{\partial^2}{\partial x^2} + A_{11} \frac{\partial^2}{\partial y^2}, \\ k_{23} &= -(B_{12} + 2B_{66}) \frac{\partial^3}{\partial x^2 \partial y} - B_{11} \frac{\partial^3}{\partial y^3}, \\ k_{24} &= k_{15}, \quad k_{25} = B_{66}^s \frac{\partial^2}{\partial x^2} + B_{11}^s \frac{\partial^2}{\partial y^2}, \quad k_{26} = G_{13}^s \frac{\partial}{\partial y}, \\ k_{33} &= B_{11}^s \left(\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} \right) + 2(B_{12}^s + 2B_{66}^s) \frac{\partial^4}{\partial x^2 \partial y^2}, \\ k_{34} &= -D_{11}^s \frac{\partial^3}{\partial x^3} - (D_{12}^s + 2D_{66}^s) \frac{\partial^4}{\partial x \partial y^2}, \\ k_{35} &= -(D_{12}^s + 2D_{66}^s) \frac{\partial^3}{\partial x^2 \partial y} - D_{11}^s \frac{\partial^3}{\partial y^3}, \\ k_{36} &= -H_{13}^s \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right), \quad k_{44} = F_{11}^s \frac{\partial^2}{\partial x^2} + F_{66}^s \frac{\partial^2}{\partial y^2} - A_{44}, \\ k_{45} &= (F_{12}^s + F_{66}^s) \frac{\partial^2}{\partial x \partial y}, \quad k_{46} = (J_{13}^s - A_{44}) \frac{\partial}{\partial x}, \\ k_{55} &= F_{66}^s \frac{\partial^2}{\partial x^2} + F_{11}^s \frac{\partial^2}{\partial y^2} - A_{44}, \quad k_{56} = (J_{13}^s - A_{44}) \frac{\partial}{\partial y}, \\ k_{66} &= K_{33}^s - A_{44} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right). \end{aligned}$$

7.2. Appendix B

The elements of the symmetric matrix $[K]$ presented in Eq. (21) are given by

$$\begin{aligned} K_{11} &= \bar{\xi}^2 A_{11} + \bar{\eta}^2 A_{66}, \quad K_{12} = \bar{\xi} \bar{\eta} (A_{12} + A_{66}), \\ K_{13} &= -\bar{\xi} [\bar{\xi}^2 B_{11} + \bar{\eta}^2 (B_{12} + 2B_{66})], \\ K_{14} &= \bar{\xi}^2 B_{11}^s + \bar{\eta}^2 B_{66}^s, \\ P_{15} &= \bar{\xi} \bar{\eta} (B_{12}^s + B_{66}^s), \quad P_{16} = -\bar{\xi} G_{13}^s, \\ P_{22} &= \bar{\xi}^2 A_{66} + \bar{\eta}^2 A_{11}, \\ P_{23} &= -\bar{\eta} [\bar{\xi}^2 (B_{12} + 2B_{66}) + \bar{\eta}^2 B_{11}], \\ k_{24} &= K_{15}, \quad K_{25} = \bar{\xi}^2 B_{66}^s + \bar{\eta}^2 B_{12}^s, \\ K_{26} &= -\bar{\eta} G_{13}^s, \\ K_{33} &= (\bar{\xi}^4 + \bar{\eta}^4) B_{66}^s + 2\bar{\xi}^2 \bar{\eta}^2 (B_{12}^s + B_{66}^s), \\ K_{34} &= -\bar{\xi} [\bar{\xi}^2 D_{11}^s + \bar{\eta}^2 (D_{12}^s + 2D_{66}^s)], \\ K_{35} &= -\bar{\eta} [\bar{\xi}^2 (D_{12}^s + 2D_{66}^s) + \bar{\eta}^2 D_{11}^s], \\ K_{36} &= (\bar{\xi}^2 + \bar{\eta}^2) H_{13}^s, \\ K_{44} &= \bar{\xi}^2 F_{11}^s + \bar{\eta}^2 F_{66}^s + A_{44}, \\ K_{45} &= \bar{\xi} \bar{\eta} (F_{12}^s + F_{66}^s), \quad K_{46} = \bar{\xi} (A_{44} - J_{13}^s), \\ K_{55} &= \bar{\xi}^2 F_{66}^s + \bar{\eta}^2 F_{11}^s + A_{44}, \quad K_{56} = \bar{\eta} (A_{44} - J_{13}^s), \\ K_{66} &= (\bar{\xi}^2 + \bar{\eta}^2) A_{44} + P_{33}^s, \end{aligned}$$

where $\bar{\xi} = \xi/h$ and $\bar{\eta} = \eta/h$.

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