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ПРОГНОЗИРОВАНИЕ КОГЕРЕНТНЫХ РАЗРЫВОВ ВОЛАТИЛЬНОСТИ

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АННОТАЦИЯ
Разработана методика долгосрочного (до нескольких месяцев) прогнозирования разворотной динамики волатильности с использованием свойств длинной памяти финансовых временных рядов. Предложенный в [1] алгоритм вычисления фрактальной размерности через покрытие предфракталами используется для декомпозиции волатильности на удельную $\overset{0}{A}(t)$ и структурную $\overset{\mu}{H}(t)$. Предложены модели динамических компонент волатильности, способные предсказывать длинные восходящие в ней тренды. Для проверки статистической значимости прогнозов введены функции оценки условных и безусловных вероятностей для наблюдаемых и прогнозируемых компонент. Наши результаты могут быть использованы для предсказания точек перехода рынка в нестабильное состояние.
Ключевые слова: фондовый рынок; ценовой риск; фрактальная размерность; крахи рынка; ARCH-GARCH модель; модели волатильности как амплитуды; многомасштабная волатильность; развороты волатильности; технический анализ.

FORECASTING COHERENT VOLATILITY BREAKOUTS

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ABSTRACT
The paper develops an algorithm for making long-term (up to three months ahead) predictions of volatility reversals based on long memory properties of financial time series. The approach for computing fractal dimension using sequence of the minimal covers with decreasing scale (proposed in [1]) is used to decompose volatility into two dynamic components: specific $\overset{0}{A}(t)$ and structural $\overset{\mu}{H}(t)$. We introduce two separate models for $\overset{0}{A}(t)$ and $\overset{\mu}{H}(t)$, based on different principles and capable of catching long uptrends in volatility. To test statistical significance of its abilities we introduce several estimators of conditional and unconditional probabilities of reversals in observed and predicted dynamic components of volatility. Our results could be used for forecasting points of market transition to an unstable state.
Keywords: stock market; price risk; fractal dimension; market crash; ARCH-GARCH; range-based volatility models; multi-scale volatility; volatility reversals; technical analysis.

1. INTRODUCTION

At least from early 1950-s volatility, taken as a proxy for price risk, is being at the core of financial theory and practice. There is still significant gap between the latter and the former in defining the very notion of volatility: while most practitioners admit that volatility is the range between maximum and minimum price, theoretical finance holds volatility merely as the variance (or standard deviation) of returns. This approach is rooted in the seminal paper authored by Bachelier [2], the father of random walks theory (RWT). His findings were independently re-discovered and justified by Osborne [3]. General RWT line of thoughts was adopted by Markowitz, who suggested to use variance [4] and later — semivariance [5] of returns as a proxy for risk. Soon ideas of Bachelier and Osborne became central to capital asset [6], and option pricing theories (e.g. Black-Sholes model, [7]). Currently there is vast amount of literature which undermines random walks approach from empirical side, and another, of comparable size, which supports it, predominantly from theoretical positions.

One of the earliest works to raise the question of the adequacy of Bachelier' model assumptions, became Fama's empirical study [8] of whether stock returns follow normal or stable Paretian distribution (the latter suggested by Fama's supervisor Mandelbrot in [9] and [10]). Author describes the following stylized fact: although the changes in prices (yields) show no autocorrelation (as prescribed by RWT), there is still very significant autocorrelation of squared and absolute price changes. Paper by Fama had generated significant interest to empirical facts about observed time series and its deviations from what was predicted by RWT, including volatility clustering. This eventually led to Engle's propositions of a new class of stochastic processes — autoregressive conditional heteroscedasticity (ARCH) — which accounted for mentioned stylized fact about variance [11]. Engle's contribution has spawned a whole new area in finance, whithin which many ARCH-type models were proposed, and above all — dozens of modifications of the conditional variance equation. The most significant contribution in this area belongs to Bollerslev, who generalized ARCH to GARCH in [12], and Nelson, who developed exponential GARCH — EGARCH [13].

After stylized facts about the variable structure of volatility on different time scales was documented [14], another subclass of ARCH-type processes — HARCH — was proposed in [15], in which volatility-variance was modeled simultaneously for multiple time scales. The authors justified their approach by stating that as market participants differ by their preferred investment horizon, volatility parameters at different time scales can and should differ too.

In parallel with the school of thoughts which regarded volatility as variance, another view on the financial markets has evolved, based on the so-called «Dow theory». Initially this approach took into account only closing prices (comp. Bachelier, for which a financial time series were the sequence of returns). However, very soon practitioners who followed tradition to capture price data in the «open-high-low-close» format have discovered a number of analytical heuristics based solely upon the use of highs and lows data. These heuristics (connected with «Dow theory") were called «technical analysis». It is believed that the first collection of these heuristics in some semblance of the theory were implemented by Edwards and Magee ({Edwards: 1948ve}). Subsequently, their work has been reprinted many times, and the «theory» has acquired a number of offshoots and apocrypha. Despite the widespread use of it in market practice, it has long history of being criticized by academicians.

In Edwards and Magee version of technical analysis volatility was understood as the range between extreme highs and lows for the period. Apparently, the first attempts to draw the attention of the academic community on the importance of the ranges have been made in 1980 by Parkinson ({Parkinson: 1980uy}) and Garman ({Garman: 1980wn}). Parkinson shows that even under random walk assumption, price ranges (called «extreme values") should be more effective as estimators of volatility than squared or absolute returns. These conclusions were supported by the following stylized facts: autocorrelation of price ranges is significantly higher than autocorrelation of squared and absolute returns.

Originally Parkinson and Garman-Klass volatility estimators received relatively limited attention of academic environment (among few papers developing their ideas before 2006 ([19],

[20] and could be named). In recent years there has been a surge of interest to modeling volatility as a range, see e.g. A brief overview of models of this type is shown in; more detailed introduction is given in. It is noteworthy that almost all proposed models of this kind are also of GARCH type. In all mentioned papers volatility is modeled and predicted only on one, more or less arbitrary selected scale; the possibility of a complex structure of volatility across time scales is ignored. More to it, ARCH/GARCH-type model would be predominantely «predicting the past», i.e. forecasting low volatility when observed volatility is low and high volatility, when the opposite is observed.

The rest of the paper is organized as follows. Next session introduces an algorithm to unbundle structural (scale-specific) dynamic component of multiscale volatility from time-specific dynamic component using fractal measure introduced by Dubovikov in [1]. Sections 3 introduces two separate models for these dynamic components, addressing mentioned issues (i.e. ability to predict volatility reversals on many scales simultaneously). Several estimators of conditional and unconditional probabilities of reversals in observed and predicted dynamic components of volatility are also introduced in Section 3 and subsequently used to backtest suggested algorithm on several major assets. Section 4 concludes.

2. DYNAMIC VOLATILITY COMPONENTS

Our research builds upon and further extends results reported in [16], [17], [18]. We use the following notation, adopted in previous works. Let $P(t)$ denote a price time-series, considered on the interval $[t-\delta_c, t]$, where δ_c is called characteristic scale of $P(t)$. Let δ_k be aliquot divisor of δ_c, i.e. $\delta_k = \frac{\delta_c}{k}$. Let $\Delta_1, .., \Delta_k$ be the partition of $[t-\delta_c, t]$ to k segments $[t-\delta_c, t]$. Then $A_P(\Delta_i) = \max(P, \Delta_i) - \min(P, \Delta_i)$ would be the amplitude of the function $P(t)$ on interval i and $V_P(\delta_k) = \sum_{i=1}^{k} A_P(\Delta_i)$ would be variation of $P(t)$ on the interval $[t-\delta_c, t]$.

Important to note, that with $\delta_k \to 0$ (hence $k \to \infty$) $V_P(\delta_k)$ becomes directly related to the fractal dimension of $P(t)$, namely:

$$V_p\left(\delta_k\right) \sim \delta_k^{-\mu} \tag{1}$$

Moreover,

$$\mu = D - 1 \tag{2}$$

where D is fractal dimension of $P(t)$. If $\delta_0 = 0 \cdot$ then parameter μ could be estimated using regression of the form

$$\log V(\delta_k) = \alpha - \mu \log(\delta_k) \tag{3}$$

wherein the estimate of μ would not depend upon log base in (3) (unlike α estimate). To make μ interpretation more intuitive it is advisable to subtract it from unity

$$H^{\mu} = 1 - \mu \tag{3.1}$$

and let

$$\alpha = A^{0}(t) \tag{3.2}$$

According to results previously reported in [16], [18], regression (3) have extremely high determination coefficient (almost equal to 1), which makes estimates of $H^{\mu}(t)$ and $A^{0}(t)$ virtually independent of the choice of divisors for δ_c. Interpretation of $H^{\mu}(t)$ and $A^{0}(t)$ is as follows. $H^{\mu}(t)$ shows expected change in the average range across scales (i.e. how, for example, average volatility of the daily data would differ from the average volatility of the weekly data). In other words, it shows how the scale factor affects the average volatility. $A^{0}(t)$ in turn, according to the standard interpretation of the regression would show expected volatility (average range) of the series, when the scale factor is zero, that is, the volatility of the «unit» scale. We propose to call $A^{0}(t)$ specific and $H^{\mu}(t)$ — structural volatility.

In this research we use weekly time series, with $\delta_c = 32$. Regression (3) could be continiously estimated on $[t - \delta_c, t]$, which would result in series of dynamic variables $H^{\mu}(t)$ and $A^{0}(t)$. In [16] it was shown that the dynamics of these variables is determined by the behavior of the price series $P(t)$. Namely, the following types of price behavior could be defined:

1. Trend (both upward and downward), when there is a significant price change on the scale of characteristic order. Volatility rises sharply on intervals with trend. It is shown that as a rule the beginning of the trend is accompanied by a fall in $H^{\mu}(t)$, and maintaining trend condition requires relatively small values of this function.
2. Flat, or sideways move, when the price varies little on scales comparable to the characteristic one. Empirically, entry into flat is often accompanied by an increase of $H^{\mu}(t)$, and maintaining flat condition requires relatively high values of this function.
3. Walks, an intermediate state between the trend and flat.

Further, the following relation holds:

$$\log_{\delta_c}[V(\delta_c)] \approx \alpha - \mu \tag{4}$$

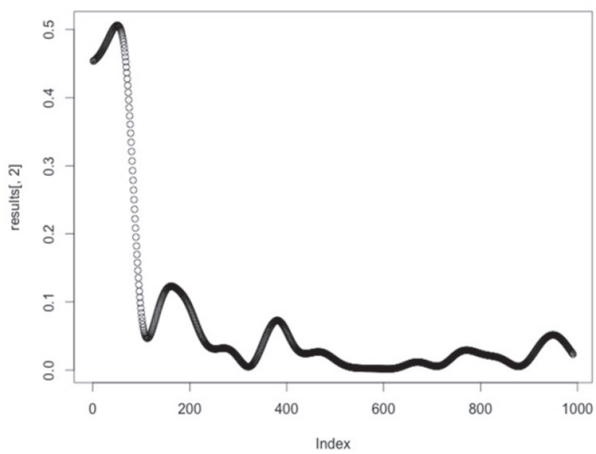
From (4) it follows that the most pronounced pattern of the market entering the unsteady state period should be when the function $\alpha(t)$ rises, and $\mu(t)$ drops sharply (i.e. $H^{\mu}(t)$ and $A^{0}(t)$ rise simultaneously). We propose to call such period of financial time the coherent breaks. As the coherent break corresponds to the most significant price changes, if some model for $H^{\mu}(t)$ and $A^{0}(t)$ would be capable of forecasting these parameters, it would to a certain extent allow to predict periods of market unsteady states.

3. FORECASTING COHERENT BREAKOUTS

To model $H^{\mu}(t)$ we use regression analysis. The form of the regression is based on the fact that this function has a fairly pronounced quasi-cyclic structure, i.e. its evaluation function can be obtained using the Fourier harmonics. Fitting the model is done as follows: first, regression of the form

$$H^{\mu}(t) = c_1 + c_2 \sin(\omega t) + c_3 \cos(\omega t) \quad (5)$$

is fitted for all frequencies ω, taken with step $\Delta = 0.0001$ on the $[0, 0.1]$. Second, local maximas of the function R2 (ω) are considered. Empirically, for any given t there would also be 2–3 local extremes, which are clearly distinguishable (typical chart is given on the Fig.)



Function R2 (ω) for S&P-500 Index (daily data)

The model is built is as follows: regression (5) is constructed with a frequency corresponding to the maximum value of the determination coefficient. Then, the evaluation function is subtracted from the original. For the residual function regression (5) is considered again. The procedure continues as long as the adapted coefficient of determination increases.

As a result, the evaluation function $\hat{H}^{\mu}(t)$ is represented in the form:

$$\hat{H}^{\mu}(t) = c + \sum_{i=1}^{k} [a_i \sin(\omega_i t) + b_i \cos(\omega_i t)] \quad (6)$$

Usually, the coefficient of determination reached an average of about 0.7.

As for the $A^{0}(t)$ function, the most convenient way for its prediction is technical analysis indicator Zig-Zag, ([19], [20]), which is essentially a piecewise linear approximation of the function. Trend intervals of $A^{0}(t)$ are approximated with straight lines. The direction of the piecewise linear trend is changing if the trend in the function being evaluated reverses at a value greater than a certain value, which is a parameter of the function Zig-Zag. As the angle of the left segment of the function may change every time when new data appears on the left side of the chart, Zig-Zag is recalculated at each step of our backtest.

When backtesting the model described, we have focused on identifying capability of the model to forecast long spans of coherent breaks. Estimators of $H^{\mu}(t)$ and $A^{0}(t)$ were built continuously for window of 480 observations moving along P (t) with step of 4 observations (which accounts for approximately one month in case of weekly sampled data). The object of the test was to measure the relative share of predicted dynamics for $H^{\mu}(t)$ and $A^{0}(t)$ matched to observed dynamics. We tested for ability of the model to forecast areas of coherent breaks for horizon of 8, 12, 16 weeks. To do that we considered following functions:

$$K(4,8) = ant\{(sign[\alpha(t+4) - \alpha(t)] + sign[\alpha(t+8) - \alpha(t+4)] + 2) / 4\} \times ant\{sign[\mu(t) - \mu(t+4)] + \\ + sign[\mu(t+4) - \mu(t+8)] + 2) / 4\}$$

$$K(4,12) = ant\{(sign[\alpha(t+4) - \alpha(t)] + sign[\alpha(t+8) - \alpha(t+4)] + sign[\alpha(t+12) - \alpha(t+8)] + \\ +3) / 6\} * ant\{sign[\mu(t) - \mu(t+4)] + sign[\mu(t+4) - \mu(t+8)] + \\ +sign[\mu(t+8) - \mu(t+12)] + 3) / 6\}$$

$$K(4,16) = ant\{(sign[\alpha(t+4) - \alpha(t)] + sign[\alpha(t+8) - \alpha(t+4)] + sign[\alpha(t+12) - \alpha(t+8)] + \\ +sign[\alpha(t+16) - \alpha(t+12)] + 4) / 8\} * ant\{sign[\mu(t) - \mu(t+4)] + sign[\mu(t+4) - \mu(t+8)] + \\ +sign[\mu(t+8) - \mu(t+12)] + sign[\mu(t+12) - \mu(t+16)] + 4) / 8\}$$

For both observed and predicted $H^{\mu}(t)$ and $A^{0}(t)$ functions $K(4,8)$, $K(4,12)$, $K(4,16)$ would be equal to 1, when coherent break of length 8,12,16 is observed or forecasted, respectively, and zero in all other cases.

Then we built functions $L(4,i) = K^h(4,i)K^f(4,i)$ $(i = 8,12,16)$ where $K^h(4,i)$ and $K^f(4,i)$ are defined for historicas and forecasted data, respectively. Then we compare unconditional statistic probability of coherent breaks

$$P(4,i) = \frac{\sum_{n=1}^{N}[K^{\mathrm{h}}(4,\mathrm{i})]}{N}$$

where N is the total number of observations of historical K (4, i); with unconditional probability, i.e. probability of coherent break conditional on the fact, that the break (i.e. coherence in $H^{\mu}(t)$ and $A^{0}(t)$ change) was predicted:

$$P_f(4,i) = \frac{\sum_{n=1}^{N}[\mathrm{L}(4,\mathrm{i})]}{\sum_{n=1}^{N}[K^{\mathrm{f}}(4,\mathrm{i})]}$$

Typical results are listed in *Table*.

Financial time series	Unconditional frequency of coherent breaks of length l			Unconditional frequency of coherent breaks of length l		
	l =8	*l* =12	*l* =16	*l* =8	*l* =12	*l* =16
S&P500	0.063	0.024	0.004	0.103	0.038	0.000
MICROSOFT	0.049	0.016	0.000	0.090	0.000	0.000
AMAZON	0.131	0.066	0.016	0.150	0.111	0.000

Similar results hold for other financial assets.

4. CONCLUSIONS

As seen from Table 1, the prediction of the coherent break in most cases significantly increases the probability of its occurrence. This effect is most pronounced for a horizon of 8 and 12 weeks, while at the same time for 16 weeks horizon forecasting opportunities are vanishing. This is because the coherent breaks of such length are extremely rare. At the same time, it should be noted that in almost all cases, when there was a coherent break of the length more than 12 weeks, the model predicted the break of up to 12 weeks length. Thus, the presence of the prediction of coherent break could be accounted for observed risk factor of the market transition into an unsteady state.

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