Section 2. Mathematics

Dergachev Victor M.,

associate professor, candidate physical. mat. sciences.

Moscow polytechnical university,

Lelyavin Sergey N.,

associate professor, candidate, physical. mat. sciences

The Moscow state technical university of N.E. Bauman

E-mail: lel@internets.ru

CREATION OF THE INFINITE ALGEBRAIC K – FUNCTORS OF THE P. II

Abstract: At the solution of various problems of algebraic topology it is useful to have a design of the classifying space [1, 103–109].

In language of simplicial sets creation of a functor from category of nilpotent subalgebras is carried out to category of conditionally minimum flags. Theorems are proved: about existence of a minimum flag in a set K – admissible flags, about nilpotent subalgebras and their corollary.

Keywords: simplex, algebra, subalgebras, flag, module, homomorphism, isomorphism.

Let A – an associative ring with identity, A^N – the free A – the module over a ring A , then tops in $G_N(A)$ we will call various decomposition A^N – the free N – the measuring module in the ordered direct sum of projective submodules: $A^N = P \oplus Q$, where P, Q – projective submodules of the module A^N , $(P, Q) = a$ we will designate as tops – there are zero-dimensional simplexes in sets $G_N(A)$ and $V_{N,K}(A)$ [2, 81–83].

Let's use some concepts and the definitions given in [5, 13–16], which will be necessary at the proof of the theorem formulated in work [3, 567–572].

Theorem 1. The set D_k^* has a minimum flag.

Proof

We will deliver to each flag $F \in D_k^*$ in compliance integer function $f(e_1, \dots, e_k, P_1, \dots, P_{l+1})$ as follows: if N – length of a flag F and $e_j \in F_i, e_j \notin F_{i+1}$, then, $f(e_j) = N - i$ ($A^N = F_0$) similarly if $P_k \in F_i, P_k \notin F_{i+1}$,

that $f(P_k) = N - i$. On the contrary, on each such function it is possible to restore the flag F defining it unambiguously.

Let $N_1 = \max_{\substack{i=1, \dots, k \\ j=1, \dots, l+1}} \{f(e_i), f(P_j)\}$, then the arbitriest

module, $n = 0, 1, \dots, N_1$ of a flag F

is defined so: $F_{N_1-n} = \bigoplus_{i,j} \{e_i, P_j : f(e_i) \leq n, f(P_j) \leq n\}$.

Let's define on a set of all integer functions $\tilde{D}_k^*$ of flags $F \in D_k^*$ integer function:

$f_{inf}(e_i) = \inf_{f \in \tilde{D}_k^*} f(e_i), f_{inf}(P_j) = \inf_{f \in \tilde{D}_k^*} f(P_j)$. Let's designate

the flag corresponding to the constructed function f_{inf} through we F_{inf} . Will show that $F_{inf} \in D_k^*$.

Let's consider the arbitriest vector e_j , we will take such integer function f , as $f(e_j) = f_{inf}(e_j)$ and $F = \{A^N = F_0 \supseteq F_1 \supseteq \dots \supseteq F_m \supseteq 0\}$ – a flag corresponding to function f . Let $f(e_j) = i$, then $e_j \in F_m, e_j \notin F_{m-i+1}$. If shows $\{e_k\}$ and P_n the arbitriest modules from F_{m-i+1} , then $f_{inf}(e_k) \leq f(e_k) < i, (P_n) \leq f(e_n)$. This

reasoning that $e_j \in F_{N_2-i}, e_j \notin F_{N_2-i+1}$ in a flag $F_{inf} = \{A^N \supseteq \dots \supseteq F_{N_2-i} \supseteq F_{N_2-i+1} \supseteq \dots \supseteq F_{N_2} \supseteq 0\}$, the similar reasoning is fair for the arbitrariness of module $P_k, k=1, \dots, l+1$, then we $F_{inf} \in D_k^*$.

Will show minimality of a flag F_{inf} in D_k^* .

Let $F_{inf} = \{A^N \supseteq \dots \supseteq F_i \supseteq F_{i+1} \supseteq \dots \supseteq 0\}$ the flag be not minimum, i.e. it is possible to delete from it some module $F_i \neq i$ so that the received flag $F' = \{A^N \supseteq \dots \supseteq F_i \supseteq F_{i+1} \supseteq \dots \supseteq 0\}$ will be an element of a set D_k^* . Will designate the function corresponding to a flag F' through f' , then function is not minimum. Really, on elements $e_j, P_s \in F_k, e_j, P_s \notin F_{k+1}$ in a flag $F_{inf}, k > i, f'(e_j) < f_{inf}(e_j), f'(P_s) < f_{inf}(P_s)$. The Received contradiction proves minimality of a flag F_{inf} .

Remark 1. If the module Q in flags $F \in D_k^*$ is fixed, then the flag F_{inf} is, apparently, the single minimum flag for a set D_k^* .

Let's consider some definitions according to work [5, 13–16].

Definition 1. Let (P_1, Q) and (P_2, Q) tops in $G_N(A)$, then exists φ – a homomorphism A - modules: $\varphi: P_1 \rightarrow Q: P_1 \oplus Q \rightarrow P_2 \oplus Q$ and any element $x \in P_1$ passes into an element $x + \varphi(x) \in P_2$, then the module P_2 is called a module projection P_1 on display φ along the module Q .

Definition 2. Linearly serially finite sets of tops $(a_0, a_1, \dots, a_n)$ in $G_N(A)$ and $(b_0, b_1, \dots, b_n)$ in $V_{N,K}(A)$ is called nilpotent and K – nilpotent respectively [4, 20–22]. The nilpotent algebra $NA(a_0, a_1, \dots, a_n)$ is generated by operators $\varphi_i, \psi_i, i=1, \dots, n$, which define display: modules P_0, Q_0 tops a_0 in modules P_i, Q_i tops a_i .

Let σ substitution at a set of tops $a_0, a_1, \dots, a_n$, interchanging the position only of two tops a_0, a_i , similarly through σ we will designate substitution at a set of tops $b_0, b_1, \dots, b_n$, interchanging the position only of two tops b_0, b_i , then theorems [3, 567–572] which proofs we will provide are fair.

Theorem 2. Let $NA(a_0, a_1, \dots, a_n)$ a nilpotent subalgebra for a nilpotent set of tops $(a_0, a_1, \dots, a_n)$ from $G_N(A)$ and Σ_{n+1} a permutation group $n+1$ of ele-

ments, then for any substitution $\sigma \in \Sigma_{n+1}$ equality is fair: $NA(a_0, a_1, \dots, a_n) = NA(a_{\sigma(0)}, a_{\sigma(1)}, \dots, a_{\sigma(n)})$.

Theorem 3. Let $NA_k(b_0, b_1, \dots, b_n)$ K – a nilpotent subalgebra for K – a nilpotent set of tops from $V_{N,K}(A)$ and Σ_{n+1} a permutation group $n+1$ of elements, then for any substitution $\sigma \in \Sigma_{n+1}$ equality is fair: $NA_k(b_0, b_1, \dots, b_n) = NA_k(b_{\sigma(0)}, b_{\sigma(1)}, \dots, b_{\sigma(n)})$.

Proofs

Let $\varphi'_i: P_i \rightarrow Q_i$ a homomorphism of modules that the module P_0 is a module P_i projection along the module Q_i , similarly $\psi'_i: Q_i \rightarrow P_i$ – a homomorphism of modules that the module Q_0 is a module projection Q_i along the module P_i at this homomorphism, then by analogy:

$\varphi'_j: P_i \rightarrow Q_i, \psi'_j: Q_i \rightarrow P_i, j=1, \dots, n, j \neq i$ are defined as such homomorphisms of modules, that the module P_j is a module projection P_i along the module Q_i , at a homomorphism φ'_j , and the module Q_j – a module Q_i projection along the module P_i at a homomorphism ψ'_j . Can easily be proved, that homomorphisms φ'_i, ψ'_i as follows are expressed through φ_i, ψ_i :

$$\varphi'_i = \left[1 - (1 + \varphi_i) \cdot (1 - \psi_i \cdot \varphi_i)^{-1} \right] \cdot (1 - \psi_i),$$

$$\psi'_i = \left[1 - (1 + \psi_i) \cdot (1 - \varphi_i \cdot \psi_i)^{-1} \right] \cdot (1 - \varphi_i).$$

It is possible to check that display of the module P_0 in the module P_j along the module Q_i on display $\varphi_i: P_0 \rightarrow Q_0$ is expressed on a formula:

$1 + (1 + \psi_i) \cdot (1 - \varphi_j \cdot \psi_i)^{-1} \cdot \varphi_j$. Display φ'_j of the module P_i in the module P_j along the module Q_i is composition of display φ'_i of the module P_i in the module P_0 along the module Q_i and display:

$1 + (1 + \psi_i) \cdot (1 - \varphi_j \cdot \psi_i)^{-1} \cdot \varphi_j$ the module P_0 in the module P_j along the module Q_i :

$$1 + \varphi'_j = \left\{ 1 + \left[1 - (1 + \varphi_i) \cdot (1 - \psi_i \cdot \varphi_i)^{-1} \right] \cdot (1 - \psi_i) \right\} \left[1 + (1 + \psi_i) \cdot (1 - \varphi_j \cdot \psi_i)^{-1} \cdot \varphi_j \right] \cdot (1 + \psi_i).$$

Display $1 + \varphi'_j$ is written down in coordinate modules P_0, Q_0 of the module $A^N = P_0 \oplus Q_0$. Action of display $1 + \psi_i$ on module elements P_0 identically. Display $1 + \psi_i$ transfers module elements Q_0 to module elements Q_i , so action of displays:

$1 + (1 + \psi_i) \cdot (1 - \varphi_j \cdot \psi_i)^{-1} \cdot \varphi_j \cdot 1 + [1 - (1 + \varphi_i) \cdot (1 - \psi_i \cdot \varphi_i)^{-1}] \cdot (1 - \psi_i)$ sets an identity mapping on the module Q_i , i.e. display φ'_j displays the module Q_i in zero.

Replacing displays φ_i, φ_j to displays ψ_i, ψ_j and ψ_i, ψ_j on φ_i, φ_j in a formula for $1 + \varphi'_j$, then

$$1 + \psi'_j = \left\{ 1 + [1 - (1 + \psi_i) \cdot (1 - \varphi_i \cdot \psi_i)^{-1}] \cdot (1 - \varphi_i) \right\} \cdot [1 + (1 + \varphi_i) \cdot (1 - \psi_j \cdot \varphi_i)^{-1} \cdot \psi_j] \cdot (1 + \varphi_i).$$

Let $NA'(a_i, \dots, a_0, \dots)$ the subalgebra generated by homomorphisms: $\varphi'_i, \varphi'_j, \psi'_i, \psi'_j, j = 1, \dots, n, j \neq i$. K – the nilpotent subalgebra $NA_k(b_0, b_1, \dots, b_n)$ is generated by operators $G_i, \varphi_i, \psi_i, i = 1, \dots, n$, who define displays of modules $P_0 = \{u_1^0, \dots, u_k^0\}, Q_0$ in modules $P_i = \{u_1^i, \dots, u_k^i\}, Q_i$. Let σ – substitution at a set of tops $(b_0, b_1, \dots, b_n)$, which rearranges in places only two tops we b_0, b_i . Let's designate through operators $\varphi'_i, \psi'_i, G'_i$ who translate modules P_i, Q_i tops b_i in modules P_0, Q_0 tops b_0 . These operators are defined similar to operators φ_i, ψ_i, G_i . Through $\varphi'_i, \psi'_j, G'_j, j = 1, \dots, n, j \neq i$ we will designate operators who define displays of modules $P_i = \{u_1^i, \dots, u_k^i\}, Q_i$ of top $b_i = (u_1^i, \dots, u_k^i, Q_i)$ in $P_j = \{u_1^j, \dots, u_k^j\}, Q_j$ modules of top $b_j = (u_1^j, \dots, u_k^j, Q_j)$. Operators $\varphi'_i, \psi'_i, \varphi'_j, \psi'_j, j = 1, \dots, n, j \neq i$ coincide with similar operators for a subalgebra $NA'(a_i, \dots, a_0, \dots)$. The operator G'_i is the difference of two operators: the operator who transfers basis $u_1^i, \dots, u_k^i$ of the module P_i to basis $u_1^0, \dots, u_k^0$ of the module P_0 and the operator $1 + \varphi'_i$, where the operator φ'_i designs the module P_i on the module P_0 along the module Q_i . It is easily possible to check that the operator G'_i says through operators φ_i, G_i, ψ_i :

$$G'_i = (1 - \varphi_i) \cdot G_i \cdot (1 + \varphi_i) - \psi_i \cdot \varphi_i \cdot (1 + G_i) \cdot (1 + \varphi_i).$$

The operator G'_i is the difference of operators: the operator of the module P_i translating basis $u_1^i, \dots, u_k^i$ in basis $u_1^j, \dots, u_k^j$ of the module P_j and the operator $1 + \varphi'_j$, where the operator φ_j designs the module P_i in the module P_j along the module Q_i , therefore he says through operators $\varphi_i, \psi_i, G_i, \varphi_j, \psi_j, G_j, j = 1, \dots, n$ on a formula: $1 + G'_j = (1 + G_j) \cdot (1 + \varphi_j) \cdot (1 - \varphi_i) \cdot (1 - G_i)$,

$$\{1 + [1 - (1 + \varphi_i) \cdot (1 - \psi_i)] \cdot (1 - \psi_i)\}^{-1} \cdot [1 + (1 + \psi_i) \cdot (1 - \varphi_j \cdot \psi_i)^{-1} \cdot \varphi_j]^{-1}.$$

Представим, операторы $\varphi'_i, \psi'_i, \varphi'_j, \psi'_j, G'_i, G'_j$ через линейные части от операторов $\varphi_i, \psi_i, \varphi_j, \psi_j, G_i, G_j, j = 1, \dots, n, j \neq i$: $\varphi'_i = -\varphi_i + \dots$, $\psi'_i = -\psi_i + \dots$, $\varphi'_j = -\varphi_i + \varphi_j + \dots$,

$$\psi'_j = -\psi_i + \psi_j + \dots, G'_i = G_i + \dots, G'_j = G'_j - G_i + \dots$$

Let's present, operators $\varphi'_i, \psi'_i, \varphi'_j, \psi'_j, G'_i, G'_j$ through the linear parts from operators $\varphi_i, \psi_i, \varphi_j, \psi_j, G_i, G_j, j = 1, \dots, n, j \neq i$: $\varphi'_i = -\varphi_i + \dots$, $\psi'_i = -\psi_i + \dots$, $\varphi'_j = -\varphi_i + \varphi_j + \dots$, $\psi'_j = -\psi_i + \psi_j + \dots$, $G'_i = G_i + \dots$, $G'_j = G'_j - G_i + \dots$

It is easy to prove that transition $\varphi_i, \psi_i, \varphi_j, \psi_j$, from forming generating a subalgebra $NA(a_0, a_1, \dots, a_n)$ to new forming $\varphi'_i, \psi'_i, \varphi'_j, \psi'_j$, sets an isomorphism of a subalgebra $NA(a_0, a_1, \dots, a_n)$, the similar statement is fair for K – a nilpotent subalgebra $NA_k(b_0, b_1, \dots, b_n)$, generated by homomorphisms $\varphi_i, \psi_i, G_i, i = 1, \dots, n$.

Corollary 1. (theorem 2.) Let the investment $(a_{i_0}, \dots, a_{i_k}) \subset (a_0, a_1, \dots, a_n)$ of nilpotent subsets of tops from $G_N(A)$, then be available an investment of nilpotent subalgebras: $NA(a_{i_0}, \dots, a_{i_k}) \subset NA(a_0, a_1, \dots, a_n)$.

Proof

Let's rearrange tops $a_0, a_1, \dots, a_n$ so that tops $a_{i_0}, \dots, a_{i_k}$ were the first in this list of tops, i.e. it is possible to assume that inclusion takes place (after a feather – numbering of tops):

$(a_0, a_1, \dots, a_k) \subset (a_0, a_1, \dots, a_k, a_{k+1}, \dots, a_n), k < n$. To each deletion $a_i, i > k$ of top in a nilpotent set of tops $a_0, a_1, \dots, a_i, \dots, a_n$ there corresponds deletion of couple of generating homomorphisms φ_i, ψ_i in nilpotent algebra $NA(a_0, a_1, \dots, a_i, \dots, a_n)$ and therefore takes place, an investment of nilpotent subalgebras:

$$NA(a_0, \dots, \hat{a}_i, \dots, a_n) \subset NA(a_0, \dots, a_i, \dots, a_n).$$

After deletion sequentially of tops $i = k + 1, \dots, n$ we will receive an investment of subalgebras: $NA(a_{i_0}, \dots, a_{i_k}) \subset NA(a_0, a_1, \dots, a_n)$ the similar corollary can be proved for K – nilpotent subalgebras:

Corollary 2. (theorem 3.) Let $(b_{i_0}, \dots, b_{i_k}) \subset (b_0, \dots, b_n)$ the investment K – nilpotent

subsets of tops from $V_{N,K}(A)$, then be fair an investment K – nilpotent subalgebras:

$$NA_k(b_{i_0}, \dots, b_{i_k}) \subset NA_k(b_0, \dots, b_n).$$

References:

1. Dergachev V. M., Lelyavin S. N. The universal object in algebraic. To the theory. Coll. A new word in science and to practice: hypotheses and approbation of results of researches. Novosibirsk: CRNS – 2014. – No. 11. – P. 103–109.
2. Dergachev V. M., Lelyavin S. N. Algebraic To – groups as homotopy groups of variety of Grassmann. Coll. Basic and applied researches. – Novosibirsk: CRNS – 2014. – No. 12. – P. 81–83.
3. Dergachev V., Lelyavin S. Universal bundle in algebraic K – theory. Australian Journal of Scientific Research, – 2014. – No. 1. (5). – (January-June). – Volume IV. – P. 567–572.
4. Dergachev V., Lelyavin S. Design to algebraic K-functors. Austrian Journal of Technical and Natural Sciences. – No. 3–4. – 2015. – (March–April). – P. 20–22.
5. Dergachev V., Lelyavin S. Creation of the infinite algebraic K – functor of the p. I. Austrian Journal of Technical and Natural Sciences. – No. 11–12. – 2017. – (November – December), – P. 13–16.