Section 3. Mathematics

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*Dergachev Victor Mikhaylovich,
associate professor, candidate physical. mat. sciences.
Moscow polytechnical university*

*Lelyavin Sergey Nikitovich,
associate professor, candidate physical. mat. Sciences
The Moscow state technical university of N.E. Bauman
E-mail: lelyavin.s@yandex.ru*

CONSTRUCTION OF INFINITE ALGEBRAIC K- THEORY OF P I

Abstract. The geometric realization of stable simplicistic complexes is considered, and the theorem that the simplicistic complex is weakly homotopically trivial is proved.

Keywords: simplex, attachment, proobraz, homotopia, module, semi-disc.

Theorem 1. There is a simplicistic attachment: $\tilde{W} : V_{k,n}(A) \otimes \gamma^3 \rightarrow V_{k,n+k}(A)$, such that $\tilde{W}(b_0 \otimes \gamma^3) = \tilde{W}(1)(\tilde{b}_0) = \hat{b}_0$, $\tilde{W}(b_i \otimes \gamma^3) = \tilde{W}(1)(\tilde{b}_i) = \hat{b}_i$, $i = 1, \dots, l$, [3, 17–20],

Where $\tilde{W}(1)$ denotes a path consisting of a composition of three one-dimensional simplexes

$$\tilde{W}_1 \cdot \tilde{W}_2 \cdot \tilde{W}_3 : \begin{pmatrix} 1_k & 1_k & 0 \\ 0 & 1_k & 0 \\ 0 & 0 & 1_{n-k} \end{pmatrix} \begin{pmatrix} 1_k & 1_k & 0 \\ -1_k & 1_k & 0 \\ 0 & 0 & 1_{n-k} \end{pmatrix} \begin{pmatrix} 1_k & 1_k & 0 \\ 0 & 1_k & 0 \\ 0 & 0 & 1_{n-k} \end{pmatrix},$$

where 1_k denotes a unit matrix of dimension k , and this path begins from top $\tilde{b}_0$, and action of a matrix product $\tilde{W}_1 \cdot \tilde{W}_2 \cdot \tilde{W}_3 = \tilde{W}(1)$ on top $\tilde{b}_0$, is understood as action of this work on the following vector from modules: $\{e_1, \dots, e_k\} \oplus \{u_1, \dots, u_k\} \oplus \{e_{k+1}, \dots, e_n\} = A^{n+k}$, where $e_1, \dots, e_k$ – basis of the module P_0 , $e_{k+1}, \dots, e_n$ – basis of the module Q_0 , $u_1, \dots, u_k$ – basis of the free module A^k , $\tilde{W} \cdot g_0(b_0) = \tilde{b}_0$, $\tilde{W} \cdot g_0(b_i) = \tilde{b}_i$, $i = 1, \dots, l$, $\tilde{W} \cdot g_0(\sigma_V^l) = \tilde{\sigma}_V^l$,

$$\tilde{W} \cdot g_3(b_0) = \hat{b}_0, \tilde{W} \cdot g_3(b_i) = \hat{b}_i, \tilde{W} \cdot g_3(\sigma_V^l) = \hat{\sigma}_V^l [1, 7–10].$$

Theorem 2. The simplicistic complex $\hat{V}(A)$ is weakly homotopically trivial: $\pi_i(\hat{V}(A)) = 0$, $i = 0, 1, 2, \dots$ (prove that the complex $\hat{V}_k(A)$ is tightened for any).

Proof

We stabilize the complex by the second compound [2, 20–22], i.e. consider $\hat{V}_k(A) = \lim_{\overleftarrow{n}} \hat{V}_{k,n}(A)$

and by virtue of **theorem 1**. There is a simplicistic attachment: $\tilde{W} : V_{k,n}(A) \otimes \gamma^3 \rightarrow V_{k,n+k}(A)$.

Above each complex $V_{k,n}(A)$ is constructed a simplicistic cone (due to homotopia $\tilde{W}$) in the complex $V_{k,n+k}(A)$, i.e. any finite sub-complex $V_{k,n}(A)$ is tightened in the complex $\hat{V}_k(A) = \lim_{\overleftarrow{n}} \hat{V}_{k,n}(A)$, then $\pi_i(\hat{V}_k(A)) = 0$, $i = 0, 1, 2, \dots$,

The theorem is proven.

Definition 1. Let be $\sigma^n = (a_0, a_1, \dots, a_n)$ a dimensional geometric simplex where $a_0, a_1, \dots, a_n$ its vertices. Through the designation $St(\sigma^n)$ of the “cup” of the

simplex σ^n , which is obtained from the simplex σ^n by ejecting one of its $(n-1)$ -dimensional faces σ^{n-1} and its interior $\sigma_0^n \subset \sigma^n$, i.e. $St(\sigma^n) = \sigma^n \setminus (\sigma_0^n \cup \sigma^{n-1})$. the “Simplistic thickening” of the cup $St(\sigma^n)$, we will denote as $St(\sigma_1^n)$, it will have the form: $St(\sigma_1^n) = \bigcup_{i,k} (\sigma_i^n \times D_+^{n-k})$, where σ_i^n – any n – the dimensional subsimplex of the “cup” $St(\sigma^n)$ and D_+^{n-k} is $(n-k)$ – the dimensional open half-disc.

Definition 2. Let $\sigma^n = \bigcup St(\sigma_1^n) = \sigma_1^n$ – “thickening” σ^n , $j: \sigma_1^n \rightarrow \sigma^n$ – the mapping of the tightening “simplistic thickening” σ_1^n into a simplex, σ^n , i.e. resulting by pulling on itself all half-faces D_+^{n-k} , of all faces $\sigma^k \subset St(\sigma_1^k)$.

Let the B -simplium complex, then $\tilde{f}$ denotes a mapping that matches each n – dimensional geometric simplex σ^n , n – dimensional simplex in the complex B .

The composition of the mappings $\tilde{f}$ and j denote via f , i.e. $f = \tilde{f} \circ j$. It sets the mapping $f: \sigma_i^n \rightarrow B$, matching the “thickening” n – dimensional geometric simplex n – dimensional simplex σ_B^n in the complex B .

One variant of defining a covering homotopia to map the simplistic sets is the following statement.

Statement Let $p: E \rightarrow B$ the mapping of the simplistic sets be such that for any simplex σ^n and $g_0: St(\sigma_1^n) \rightarrow E$ the mapping of the “thickening” of the cup $St(\sigma_1^n)$ to the simplistic set E , wherein $p \circ g_0 = f|_{St(\sigma_1^n)}$, $f|_{St(\sigma_1^n)}$ the constraint of mapping f to a subset $St(\sigma_1^n)$, exists such mapping g from σ_1^n to E : $g: \sigma_1^n \rightarrow E$, that $p \circ g_0 = f$, then for mapping p there is an exact sequence of the pair: $\dots \rightarrow \pi_i(F) \rightarrow \pi_i(E) \rightarrow \pi_i(E, F) \rightarrow \pi_{i-1}(F) \rightarrow \pi_{i-1}(E) \rightarrow \dots$, where F is the simplistic subset in E being the pattern of the marked point from E .

Remark Let $p: F \times B \rightarrow B$ – the presentation of the product of simplistic complexes: F and B in B , this case the conditions of **statement** are obviously fulfilled, i.e. in the direct product of complexes “glass” is always filled.

Proof

Let $p \circ g_0 = f|_{St(\sigma_1^n)}$, you continue displaying, g_0 , where $g_0: \sigma_1^n \rightarrow E$, until displaying $g: \sigma_1^n \rightarrow E$, such that $p \circ g_0 = f$.

Limit the display g_0 to the cup, i.e. put $g_0|_{St(\sigma^n)} = h_0$.

If we continue h_0 to such a display $h: \sigma^n \rightarrow E$, that $p \circ h = f|_{\sigma^n}$, the **remark** will be proven, as then g_0 automatically glues with h and gives the required display g .

Consider a direct product $\sigma^{n-1} \times I, I = [0, 1]$ and a mapping $\Phi: \sigma^{n-1} \times I \rightarrow \sigma^n$, which on $\sigma^{n-1} \times \{0\}$ is identical, i.e. $\Phi = Id: \sigma^{n-1} \times \{0\} \rightarrow \sigma^n$, and $\sigma^{n-1} \times \{1\}$ maps to cup $St(\sigma^n)$.

At that, display Φ is mutually unambiguous everywhere on $\sigma^{n-1} \times I$ except for set $\Gamma \times I \subset \sigma^{n-1} \times I$, where Γ – limit of simplex σ^{n-1} .

On a set $\Gamma \times I$ we define Φ as follows: if $(x, t) \in \Gamma \times I$ and $\Phi(x, t) = x$. Then $f \circ \Phi: \sigma^{n-1} \times I \rightarrow \sigma_B^n$, the mapping where σ_B^n the simplex in the complex B , is the homotopia of the mapping f_1 to the mapping f_2 where $f_1 = f \circ \Phi|_{\sigma^{n-1} \times \{0\}}: \sigma^{n-1} \times \{0\} \rightarrow \sigma_B^{n-1}$, σ_B^{n-1} is the boundary σ_B^n , $f_2 = f \circ \Phi|_{\sigma^{n-1} \times \{1\}}: \sigma^{n-1} \times \{1\} \rightarrow \sigma_B^{n-1}$.

This homotopia we need to raise in E , i.e. construct the mapping $R: \sigma^{n-1} \times I \rightarrow E$ such that $p \circ R = f \circ \Phi$. The mapping $f \circ \Phi$ is partly already raised:

1) Display $f \circ \Phi$ on set $\sigma^{n-1} \times \{1\}$ equals display $R_1 = h_0 \circ \Phi$.

2) There is already a display Γ on the edge σ^{n-1} of the face $g_0|_{\Gamma} = h_0$.

Let’s define the mapping R_1 on a set $\Gamma \times I$: if $(x, t) \in \Gamma \times I$, then $R_1(x, t) = g_0$.

The condition $p \circ R = f \circ \Phi$ to display R_1 is met because for $(x, t) \in \Gamma \times I$ we have $p \circ R_1(x, t) = p \circ g_0 = f(x)$, and in force $\Phi(x, t) = x: f \circ \Phi(x, t) = f(x)$

In addition, the display R_1 is consistent on a set of $\Gamma \times I \cap \sigma^{n-1} \times \{1\} = \Gamma \times \{1\}$.

Consider continuing the display $R_1: \Gamma \times I \rightarrow E$, before the display $R: \sigma^{n-1} \times I \rightarrow E$, so that

$p \circ R = f \circ \Phi$ and on the set $\Gamma \times I$ we have belonging to $\sigma^{n-1} \times I$ relative to display Φ , i.e.
 $R|_{\Gamma \times I} = R_1$. $\Phi(x, t) = y$.
 Let's build the following mapping $h: \sigma^n \rightarrow E$, Let $h(y) = R(y, t)$, if $y \in \Gamma$, that $h(y) = h_0(y)$,
 for any point $y \in \sigma^n, y \notin \Gamma$ there is a single section as prove a **statement**.

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