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ATOMICITY IN INJECTIVE BANACH LATTICES¹

A. G. Kusraev

*To Semën Kutateladze
on occasion of his 70th birthday*

This note is aimed to examine a Boolean valued interpretation of the concept of atomic Banach lattice and to give a complete description of the corresponding class of injective Banach lattices.

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1. Introduction

The aim of this note is to examine a Boolean valued interpretation of the concept of atomic Banach lattice and to give a complete description of the corresponding class of injective Banach lattices. Some representation and isometric classification results for general injective Banach lattices were announced in [1, 2].

Section 2 collects some needed Boolean valued representation results following [3]. In Section 3 we demonstrate that a Boolean valued interpretation of atomicity yields some “module atomicity” over a certain f -subalgebra of the center. Section 4 deals with Boolean valued Banach lattices of summable families, which turn out to be “building blocks” for general module atomic injective Banach lattices. Section 5 exposes the main results on representation and classification of injective Banach lattices with atomic Boolean valued representation, i. e. those which are atomic with respect to their natural f -module structure.

The needed information on the theory of Banach lattices can be found in [1, 5]. Recall some definitions and notation. A real Banach lattice X is said to be *injective* if, for every Banach lattice Y , every closed vector sublattice $Y_0 \subset Y$, and every positive linear operator $T_0 : Y_0 \rightarrow X$ there exists a positive linear extension $T : Y \rightarrow X$ of T_0 with $\|T_0\| = \|T\|$; see [5, Definition 3.2.3]. Equivalently, X is an injective Banach lattice if, whenever X is lattice isometrically imbedded into a Banach lattice Y , there exists a positive contractive projection from Y onto X ; one more equivalence definition states that each positive operator from X to any Banach lattice admits a norm preserving positive extension to any Banach lattice containing X as a vector sublattice, see [3, Theorem 5.10.6]. This concept was introduced by Lotz [6]; a significant advance towards the structure theory of injectives was made by Cartwright [7] and Haydon [8].

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In what follows X stands for a real Banach lattice. We denote by $\mathbb{P}(X)$ the Boolean algebra of all band projections in X . A crucial role in the theory of injective Banach lattices is played by the concept of M -projection. A band projection π in a Banach lattice X is called an M -projection if $\|x\| = \max\{\|\pi x\|, \|\pi^\perp x\|\}$ for all $x \in X$, where $\pi^\perp := I_X - \pi$. The collection $\mathbb{M}(X)$ of all M -projections in X is a subalgebra of the Boolean algebra $\mathbb{P}(X)$.

Throughout the sequel \mathbb{B} is a complete Boolean algebra with unit $\mathbb{1}$ and zero $\mathbb{0}$, while $\Lambda := \Lambda(\mathbb{B})$ is a Dedekind complete unital AM -space such that \mathbb{B} is isomorphic to $\mathbb{P}(\Lambda)$. The unit of Λ is also denoted by $\mathbb{1}$. A *partition of unity* in \mathbb{B} is a family $(b_\xi)_{\xi \in \Xi} \subset \mathbb{B}$ such that $\bigvee_{\xi \in \Xi} b_\xi = \mathbb{1}$ and $b_\xi \wedge b_\eta = \mathbb{0}$ whenever $\xi \neq \eta$. We let $:=$ denote the assignment by definition, while \mathbb{N} , \mathbb{Q} , and \mathbb{R} symbolize the naturals, the rationals, and the reals.

2. Boolean Valued Representation

Boolean valued analysis is an useful tool in studying of injective Banach lattices [9]. We need some Boolean valued representation results as presented in [3] and [25].

Applying the Transfer and Maximum Principles to the ZFC-theorem “There exists a field of reals” we find an element $\mathcal{R} \in \mathbb{V}^{(\mathbb{B})}$ for which $\llbracket \mathcal{R} \text{ is a field of reals} \rrbracket = \mathbb{1}$. We call \mathcal{R} the *reals* within $\mathbb{V}^{(\mathbb{B})}$. The following remarkable result due to Gordon [28] tells us that the interpretation of the reals in $\mathbb{V}^{(\mathbb{B})}$ is a universally complete vector lattice with the Boolean algebra of band projections isomorphic to \mathbb{B} .

Theorem 2.1. *Let \mathcal{R} be the reals within $\mathbb{V}^{(\mathbb{B})}$. Then $\mathcal{R} \downarrow$ (with the descended operations and order) is a universally complete vector lattice with a weak order unit $\mathbb{1} := 1^\wedge$. Moreover, there exists a Boolean isomorphism χ of \mathbb{B} onto $\mathbb{P}(\mathcal{R} \downarrow)$ such that the equivalences*

$$\begin{aligned} \chi(b)x = \chi(b)y &\iff b \leq \llbracket x = y \rrbracket, \\ \chi(b)x \leq \chi(b)y &\iff b \leq \llbracket x \leq y \rrbracket \end{aligned} \tag{G}$$

hold for all $x, y \in \mathcal{R} \downarrow$ and $b \in \mathbb{B}$.

◁ See [3, Theorem 2.2.4] and [25, Theorem 10.3.4]. ▷

DEFINITION 2.2. A *complete Boolean algebra of M -projections* in X is an arbitrary order complete and order closed subalgebra $\mathbb{B} \subset \mathbb{M}(X)$. A Banach lattice X is said to be \mathbb{B} -cyclic whenever it is a \mathbb{B} -cyclic Banach space with respect to a complete Boolean algebra \mathbb{B} of M -projections. If X has the Fatou and Levi properties (see [3, 5.7.2]), then $\mathbb{M}(X)$ itself is an order closed subalgebra of the complete Boolean algebra $\mathbb{P}(X)$.

DEFINITION 2.3. Let $\Lambda = \mathcal{R} \downarrow$ be the bounded part of the universally complete vector lattice $\mathcal{R} \downarrow$; i. e., Λ is the order-dense ideal in $\mathcal{R} \downarrow$ generated by the weak order unit $\mathbb{1} := 1^\wedge \in \mathcal{R} \downarrow$. Take a Banach space \mathcal{X} within $\mathbb{V}^{(\mathbb{B})}$ and put $\mathcal{X} \downarrow := \{x \in \mathcal{X} \downarrow : |x| \in \Lambda\}$. Equip $\mathcal{X} \downarrow$ with some *mixed norm* by putting $\|x\| := \|\lvert x \rvert\|_\infty$ for all $x \in \mathcal{X}$, where the order unit norm $\|\cdot\|_\infty$ is defined as $\|\lambda\|_\infty := \inf\{0 < \alpha \in \mathbb{R} : |\lambda| \leq \alpha \mathbb{1}\}$ ($\lambda \in \Lambda$). In this situation, $(\mathcal{X} \downarrow, \|\cdot\|)$ is a Banach space called the *bounded descent* of \mathcal{X} . The terms \mathbb{B} -isomorphism and \mathbb{B} -isometry mean that isomorphism or isometry under consideration commutes with the projections from \mathbb{B} , see [3, 5.8.9].

Theorem 2.4. *A bounded descent of a Banach lattice from the model $\mathbb{V}^{(\mathbb{B})}$ is a \mathbb{B} -cyclic Banach lattice. Conversely, if X is a \mathbb{B} -cyclic Banach lattice, then in the model $\mathbb{V}^{(\mathbb{B})}$ there exists up to the isometric isomorphism a unique Banach lattice \mathcal{X} whose bounded descent is isometrically \mathbb{B} -isomorphic to X . Moreover, $\mathbb{B} = \mathbb{M}(X)$ if and only if $\llbracket \text{there is no } M\text{-projection in } \mathcal{X} \text{ other than } 0 \text{ and } \mathcal{X} \rrbracket = \mathbb{1}$.*

◁ See [3, Theorem 5.9.1]. ▷

DEFINITION 2.5. The element $\mathcal{X} \in \mathbb{V}(\mathbb{B})$ from Theorem 2.1 is said to be the *Boolean-valued representation of X* .

Theorem 2.6. Let X be a Banach lattice with the complete Boolean algebra $\mathbb{B} = \mathbb{M}(X)$ of M -projections, Λ be a Dedekind complete unital AM -space such that $\mathbb{P}(\Lambda)$ is isomorphic to \mathbb{B} . Then the following assertions are equivalent:

- (1) X is injective.
- (2) X is lattice \mathbb{B} -isometric to the bounded descent of some AL -space from $\mathbb{V}(\mathbb{B})$.
- (3) There exists a strictly positive Maharam operator $\Phi : X \rightarrow \Lambda$ with the Levi property such that $X = L^1(\Phi)$ and $\|x\| = \|\Phi(|x|)\|_\infty$ for all $x \in X$.
- (4) There is a Λ -valued additive norm on X such that $(X, |\cdot|)$ is a Banach–Kantorovich lattice and $\|x\| = \| |x| \|_\infty$ for all $x \in X$.

◁ See [3, Theorem 5.12.5]. ▷

Theorem 2.7. Suppose that X is a Banach lattice and \mathcal{X} is the completion of the metric space X^\wedge within $\mathbb{V}(\mathbb{B})$. Then $\llbracket \mathcal{X} \text{ is a Banach lattice} \rrbracket = 1$ and $\mathcal{X} \downarrow$ is lattice \mathbb{B} -isometric to $C_\#(Q, X)$ equipped with the norm $\|\varphi\| = \sup\{\|\varphi(q)\| : q \in \text{dom}(\varphi) \subset Q\}$ ($\varphi \in C_\#(Q, X)$).

◁ The proof is a due modification of [25, 11.3.8]. ▷

3. Boolean Valued Atomicity

In this section we present Boolean valued interpretation of atomicity.

DEFINITION 3.1. A positive element x of a \mathbb{B} -cyclic Banach lattice X is said to be \mathbb{B} -indecomposable or a \mathbb{B} -atom if for any pair of disjoint elements $y, z \in X_+$ with $y + z \leq x$ there exists a projection $\pi \in \mathbb{B}$ such that $\pi y = 0$ and $\pi^\perp z = 0$, while X is called \mathbb{B} -atomic if the only element of X disjoint from every \mathbb{B} -atom is the zero element.

Denote by $\text{at}(\mathcal{X})$ and $\mathbb{B}\text{-at}(X)$ the sets of atoms in \mathcal{X} and \mathbb{B} -atoms in X , respectively. Let $\text{at}_1(\mathcal{X}) := \{x \in \text{at}(\mathcal{X}) : \|x\| = 1\}$, while $\mathbb{B}\text{-at}_1(X)$ consists of all $x \in \mathbb{B}\text{-at}(X)$ with $\|\pi x\| = 1$ for all $\pi \in \mathbb{B}$. It is easy to see that $\mathbb{B}\text{-at}_1(X) = \{x \in \mathbb{B}\text{-at}(X) : |x| = 1\}$.

Proposition 3.2. Let X be a \mathbb{B} -cyclic Banach lattice identified with the bounded descent $\mathcal{X} \downarrow$ of a Banach lattice \mathcal{X} , its Boolean valued representation $\mathcal{X} \in \mathbb{V}(\mathbb{B})$. Then the following assertions hold:

- (1) $\mathbb{B}\text{-at}(X) = \text{at}(\mathcal{X}) \downarrow$.
- (2) $\mathbb{B}\text{-at}_1(X) = \text{at}_1(\mathcal{X}) \downarrow$.
- (3) X is \mathbb{B} -atomic if and only if $\llbracket \mathcal{X} \text{ is atomic} \rrbracket = 1$.

◁ (1) Observe that $x \in \text{at}(\mathcal{X})$ if and only if $x \in \mathcal{X}_+$ and for any two positive disjoint elements $x_1, x_2 \in \mathcal{X}$ with $x_1 + x_2 \leq x$ we have $x_1 = 0$ or $x_2 = 0$. Now, given $x \in \text{at}(\mathcal{X}) \downarrow$ with $y + z \leq x$ for some disjoint $y, z \in X_+$, we put $b := \llbracket y = 0 \rrbracket$ and $\pi := \chi(b)$. Since $\llbracket y \neq 0 \rightarrow z = 0 \rrbracket = 1$, we have $\llbracket y \neq 0 \rrbracket \leq \llbracket z = 0 \rrbracket$ and thus $b^* = \llbracket y \neq 0 \rrbracket \leq \llbracket z = 0 \rrbracket$. By (G) we have $\pi y = 0$ and $\pi^\perp z = \chi(b^*)z = 0$. Thus, $\text{at}(\mathcal{X}) \downarrow \subset \mathbb{B}\text{-at}(X)$ and for the converse inclusion the argument is similar.

(2) Taking into account the representation $\mathbb{B}\text{-at}_1(X) = \{x \in \mathbb{B}\text{-at}(X) : |x| = 1\}$ the claim follows easily from the following chain of equivalences:

$$\begin{aligned} x \in \text{at}_1(\mathcal{X}) \downarrow &\iff \llbracket x \in \text{at}_1(\mathcal{X}) \rrbracket = 1 \iff \llbracket x \in \text{at}(\mathcal{X}) \rrbracket = \llbracket \|x\|_{\mathcal{X}} = 1 \rrbracket = 1 \\ &\iff x \in \mathbb{B}\text{-at}(\mathcal{X}) \wedge |x| = 1 \iff x \in \mathbb{B}\text{-at}_1(X). \end{aligned}$$

(3) Let for a while \perp , $\perp\!\!\!\perp$, and $\perp\!\!\!\perp\!\!\!\perp$ stand for disjoint complements in \mathcal{X} , $X = \mathcal{X}\downarrow$, and $\mathcal{X}\downarrow$, respectively. The third claim is immediate from the first one, since the disjoint complement and the descent commute: $(A^\perp)\downarrow = (A\downarrow)^\perp$, see [3, 1.5.3]. Indeed,

$$(A^\perp)\downarrow = (A^\perp)\downarrow \cap X = (A\downarrow)^\perp \cap X = (A\downarrow \cap X)^\perp \cap X = (A\downarrow)^\perp,$$

hence putting $A := \text{at}(\mathcal{X})$ and making use of (1) we deduce that $\text{at}(\mathcal{X})^\perp = \{0\}$ within $\mathbb{V}(\mathbb{B})$ if and only if $(\mathbb{B}\text{-at}(X))^\perp = \{0\}$. \triangleright

Corollary 3.3. *Let \mathbb{B} , X , and \mathcal{X} be the same as in Proposition 3.2 and $\Lambda = \Lambda(\mathbb{B})$. Then the following assertions hold:*

(1) *$x \in X_+$ is a \mathbb{B} -atom if and only if for each $0 \leq y \leq x$ there exists $\lambda \in \Lambda_+$ with $y = \lambda x$.*

(2) *If x and y are \mathbb{B} -atoms in X_+ then there exist a pair of disjoint projections $\pi, \rho \in \mathbb{B}$ such that $\pi x \perp \pi y$, $\rho x = \lambda u$ and $\rho y = \mu u$ for some $\mu, \lambda \in \Lambda_+$ and $u = x + y$.*

\triangleleft Interpreting in the model $\mathbb{V}(\mathbb{B})$ the well-known claims corresponding to that particular case when $\mathbb{B} = \{0, I_X\}$ (see [13, Theorem 26.4.]) and using Proposition 3.2 yields the required properties. \triangleright

Definition 3.4. Given a cardinal γ , say that a \mathbb{B} -cyclic Banach lattice X is *purely (\mathbb{B}, γ) -atomic* if $X = \mathcal{D}_0^{\perp\!\!\!\perp}$ for some subset $\mathcal{D}_0 \subset \mathbb{B}\text{-at}_1(X)$ of cardinality γ and for every nonzero projection $\pi \in \mathbb{B}$ and every subset $\mathcal{D} \subset \mathbb{B}\text{-at}_1(\pi X)$ with $\pi X = \mathcal{D}^{\perp\!\!\!\perp}$ we have $\text{card}(\mathcal{D}) \geq \gamma$. Evidently, X is purely $(\{0, I_X\}, \gamma)$ -atomic if and only if X is atomic and the cardinality of $\text{at}_1(X)$ is γ or, equivalently, X is atomic and the cardinality of the set of atoms in $\mathbb{B}(X)$ equals γ . In this case we say also that X is γ -atomic.

Proposition 3.5. *A \mathbb{B} -cyclic Banach lattice X is purely (\mathbb{B}, γ) -atomic for some cardinal γ if and only if $\llbracket \gamma^\wedge \text{ is a cardinal and } \mathcal{X} \text{ is } \gamma^\wedge\text{-atomic} \rrbracket = 1$.*

\triangleleft *Sufficiency.* Assume that γ^\wedge is a cardinal and \mathcal{X} is γ^\wedge -atomic within $\mathbb{V}(\mathbb{B})$. The latter means that \mathcal{X} is atomic and $\text{card}(\text{at}_1(\mathcal{X})) = \gamma^\wedge$ within $\mathbb{V}(\mathbb{B})$. If $\Delta := \text{at}_1(\mathcal{X})$ then there exists $\phi \in \mathbb{V}(\mathbb{B})$ such that $\llbracket \phi : \gamma^\wedge \rightarrow \Delta \text{ is a bijection} \rrbracket = 1$. Note that $\phi\downarrow$ embeds γ into $\Delta\downarrow$ by [3, 1.5.8] and $\Delta\downarrow = \mathbb{B}\text{-at}_1(X)$ by Proposition 3.1. It follows that the set $\mathcal{D} := \phi\downarrow(\gamma)$ of cardinality γ is contained in $\mathbb{B}\text{-at}_1(X)$ and $X = \mathcal{D}^{\perp\!\!\!\perp}$, since $\Delta = \mathcal{D}\uparrow$ and $\mathcal{X} = \Delta^{\perp\!\!\!\perp}$. Take $b \in \mathbb{B}$ and a set \mathcal{D}' of cardinality β which is contained in $\mathbb{B}\text{-at}_1(X)$ and generates bX , i. e. $bX = (\mathcal{D}')^{\perp\!\!\!\perp}$. Then $\mathcal{D}'\uparrow$ is of cardinality $\text{card}(\beta^\wedge)$ and $\mathcal{X} = (\mathcal{D}'\uparrow)^{\perp\!\!\!\perp}$ within the relative universe $\mathbb{V}^{(\mathbb{O}, b)}$. By [3, 1.3.7] $\llbracket \gamma^\wedge = \text{card}(\gamma^\wedge) \leq \text{card}(\beta^\wedge) \leq \beta^\wedge \rrbracket = 1$ and so $\gamma \leq \beta$.

Necessity. Assume now that X is purely (\mathbb{B}, γ) -atomic and $X = \mathcal{D}^{\perp\!\!\!\perp}$ for some $\mathcal{D} \subset \mathbb{B}\text{-at}_1(X)$ of cardinality γ . Then within $\mathbb{V}(\mathbb{B})$ we have $\Delta := \mathcal{D}\uparrow \subset \text{at}_1(\mathcal{X})$, $\mathcal{X} = \Delta^{\perp\!\!\!\perp}$ and the cardinalities of Δ and γ^\wedge coincide, i. e. $\text{card}(\Delta) = \text{card}(\gamma^\wedge)$. By [3, 1.9.11] the cardinal $\text{card}(\gamma^\wedge)$ has the representation $\text{card}(\gamma^\wedge) = \text{mix}_{\alpha \leq \gamma} b_\alpha \alpha^\wedge$, where $(b_\alpha)_{\alpha \leq \gamma}$ is a partition of unity in \mathbb{B} . It follows that $b_\alpha \leq \llbracket \Delta^{\perp\!\!\!\perp} = X \text{ and } \Delta \text{ is of cardinality } \alpha^\wedge \rrbracket = 1$. If $b_\alpha \neq \mathbb{O}$ then $(b_\alpha \wedge \Delta)^{\perp\!\!\!\perp} = b_\alpha \wedge \mathcal{X}$ and $b_\alpha \wedge \Delta$ is of cardinality $\text{card}(\gamma^\wedge) = \alpha^\wedge \leq \gamma^\wedge$ in the relative universe $\mathbb{V}^{(\mathbb{O}, b_\alpha)}$. (Concerning $b_\alpha \wedge \Delta$ and $b_\alpha \wedge \mathcal{X}$ and their properties see [3, 1.3.7].) It is easy that $b_\alpha \wedge \Delta = (b_\alpha \mathcal{D})\uparrow$ and so $(b_\alpha \mathcal{D})^{\perp\!\!\!\perp} = bX$. By hypothesis X is purely (\mathbb{B}, γ) -atomic, consequently, $\alpha \geq \text{card}(b_\alpha \mathcal{D}) \geq \gamma$, so that $\alpha = \gamma$, since $\alpha \leq \gamma$ if and only if $\alpha^\wedge \leq \gamma^\wedge$. Thus, $\text{card}(\gamma^\wedge) = \gamma^\wedge$ whenever $b_\alpha \neq \mathbb{O}$ and γ^\wedge is a cardinal within $\mathbb{V}(\mathbb{B})$. \triangleright

Definition 3.6. Let γ is a cardinal. A complete Boolean algebra \mathbb{B} (as well as its Stone representation space) is said to be γ -stable whenever $\mathbb{V}(\mathbb{B}) \models \gamma^\wedge = \text{card}(\gamma^\wedge)$, i. e. $\llbracket \gamma^\wedge \text{ is a cardinal} \rrbracket = 1$. An element $b \in \mathbb{B}$ is called γ -stable if the relative Boolean algebra $[\mathbb{O}, b]$ is γ -stable, see [25, Definition 12.3.7]. Finally, say that a partition of unity $(\pi_\gamma)_{\gamma \in \Gamma}$ in \mathbb{B} with Γ a set of cardinals is *stable* if π_γ is γ -stable for all $\gamma \in \Gamma$.

Theorem 3.7. *Let X be a \mathbb{B} -atomic \mathbb{B} -cyclic Banach lattice. There exist a set of cardinals Γ and a partition of unity $(\pi_\gamma)_{\gamma \in \Gamma}$ such that $\mathbb{B}_\gamma := [\mathbb{O}, \pi_\gamma]$ is γ -stable and $\pi_\gamma X$ is purely $(\mathbb{B}_\gamma, \gamma)$ -atomic for all $\gamma \in \Gamma$.*

◁ If a \mathbb{B} -cyclic Banach lattice X is \mathbb{B} -atomic then its Boolean valued representation \mathcal{X} is atomic within $\mathbb{V}^{(\mathbb{B})}$ according to Proposition 3.1. Denote $\gamma_0 := \text{card}(\text{at}_1(\mathcal{X}))$. By [3, 1.9.11] γ_0 is a mixture of some set of relatively standard cardinals. More precisely, there are nonempty set of cardinals Γ and a partition of unity $(b_\gamma)_{\gamma \in \Gamma}$ in \mathbb{B} such that $x = \text{mix}_{\gamma \in \Gamma} b_\gamma \gamma^\wedge$ and $\mathbb{V}^{(\mathbb{B}_\gamma)} \models \gamma^\wedge = \text{card}(\gamma^\wedge)$ with $\mathbb{B}_\gamma := [\mathbb{O}, b_\gamma]$ for all $\gamma \in \Gamma$. It follows that $b_\gamma \wedge \mathcal{X}$ is atomic Banach lattice and $\gamma^\wedge = \text{card}(\text{at}_1(b_\gamma \wedge \mathcal{X}))$ within $\mathbb{V}^{(\mathbb{B}_\gamma)}$. It remains to apply Proposition 3.5. ▷

4. The Banach Lattices $l^1(\Gamma, \Lambda)$ and $C_\#(Q, l^1(\Gamma))$

We now consider some special injective Banach lattices that are building blocks for the class of all \mathbb{B} -atomic injective Banach lattices. Recall that $\Lambda = \Lambda(\mathbb{B})$.

Given a non-empty set Γ , denote by $l^1(\Gamma^\wedge) \in \mathbb{V}^{(\mathbb{B})}$ the internal Banach lattice of all summable families $x := (x_\gamma)_{\gamma \in \Gamma^\wedge}$ in \mathcal{R} with the norm $\|x\|_1 := \sum_{\gamma \in \Gamma^\wedge} |x_\gamma|$.

Let $l^1(\Gamma, \Lambda)$ stand for the vector space of all order summable families in Λ , i.e.

$$l^1(\Gamma, \Lambda) := \left\{ \mathbf{x} : \Gamma \rightarrow \Lambda : \|\mathbf{x}\|_1 := \sum_{\gamma \in \Gamma} |\mathbf{x}(\gamma)| \in \Lambda \right\}.$$

The order on $l^1(\Gamma, \Lambda)$ is defined by letting $\mathbf{x} \leq \mathbf{y}$ if and only if $\mathbf{x}(\gamma) \leq \mathbf{y}(\gamma)$ for all $\gamma \in \Gamma$. Evidently, $l^1(\Gamma, \Lambda)$ is an order ideal of the Dedekind complete vector lattice Λ^Γ , hence so is $l^1(\Gamma, \Lambda)$. Moreover, $l^1(\Gamma, \Lambda)$ equipped with the norm $\|\mathbf{x}\| := \|\|\mathbf{x}\|_1\|_\infty$ ($\mathbf{x} \in l^1(\Gamma, \Lambda)$) is a \mathbb{B} -cyclic Banach lattice, since $\mathbb{B} = \mathbb{B}(\Lambda)$.

Proposition 4.1. *$l^1(\Gamma^\wedge)$ is a Boolean valued representation of $l^1(\Gamma, \Lambda)$ and thus $l^1(\Gamma, \Lambda)$ and $l^1(\Gamma^\wedge) \downarrow$ are lattice \mathbb{B} -isometric.*

◁ Straightforward verification shows that $l^1(\Gamma, \Lambda)$ is a Banach f -module over Λ , see [3, Definitions 2.11.1 and 5.7.1]. The modified ascent mapping $\mathbf{x} \mapsto \mathbf{x}^\uparrow$ is a bijection from $(\mathcal{R} \downarrow)^\Gamma$ onto $(\mathcal{R}^{\Gamma^\wedge}) \downarrow$, see [3, 1.5.9]. It follows from [3, 2.4.7] that $\|\cdot\|_1$ is the bounded descent of $\|\cdot\|_1$ and hence $\mathbf{x} \in l^1(\Gamma, \Lambda)$ if and only if $\|\mathbf{x}^\uparrow\|_1 \in l^1(\Gamma^\wedge) = \mathbb{1}$. Moreover, in this event $\|\|\mathbf{x}\|_1\| = \|\mathbf{x}^\uparrow\|_1 = \mathbb{1}$ so that the modified descent induces an isometric bijection between $l^1(\Gamma, \Lambda)$ and $(l^1(\Gamma^\wedge)) \downarrow$. Making use of the definition of modified descent it can be easily checked that this bijection is Λ -linear and order preserving. ▷

Proposition 4.2. *The Banach lattice $l^1(\Gamma, \Lambda)$ is \mathbb{B} -atomic and injective with $\mathbb{M}(X)$ isomorphic to \mathbb{B} . Moreover, $l^1(\Gamma, \Lambda)$ is purely (\mathbb{B}, γ) -atomic if and only if $\|\gamma^\wedge = \text{card}(\Gamma^\wedge)\| = \mathbb{1}$.*

◁ By Theorem 2.6 (2) and Propositions 3.2 and 4.1 X is injective with $\mathbb{M}(X) \simeq \mathbb{B}$ and \mathbb{B} -atomic. The second part follows from Propositions 3.5 and 4.1, since $l^1(\Gamma^\wedge)$ is $\text{card}(\Gamma^\wedge)$ -atomic within $\mathbb{V}^{(\mathbb{B})}$. ▷

Proposition 4.3. *The norm completion of \mathbb{R}^\wedge -normed space $l^1(\Gamma)^\wedge$ within $\mathbb{V}^{(\mathbb{B})}$ is a Banach lattice which is lattice isometric to the internal Banach lattice $l^1(\Gamma^\wedge)$.*

◁ Denote by \mathcal{L}_1 the completion of $l^1(\Gamma)^\wedge$ inside $\mathbb{V}^{(\mathbb{B})}$. Let A be the set of all norm-one atoms in $l^1(\Gamma)$ which is of course bijective with Γ . Then A^\wedge and Γ^\wedge are also bijective and A^\wedge can be considered as the set of all norm-one atoms in $l^1(\Gamma^\wedge)$. Denote by $\mathbb{Q}\text{-lin}(A)$ the set of all linear combinations of the members of A with rational coefficients. Then by [12, 8.4.10] we have $(\mathbb{Q}\text{-lin}(A))^\wedge = \mathbb{Q}^\wedge\text{-lin}(A^\wedge)$. Clearly, $\mathbb{Q}^\wedge\text{-lin}(A^\wedge)$ is a dense sublattice in $l^1(\Gamma^\wedge)$,

while $(\mathbb{Q}\text{-lin}(A))^\wedge$ is a dense sublattice in $l^1(\Gamma)^\wedge$ and thus in \mathcal{L}_1 , since $\mathbb{Q}\text{-lin}(A)$ is dense in $l^1(\Gamma)$. Moreover, the norms induced in $(\mathbb{Q}\text{-lin}(A))^\wedge$ by $l^1(\Gamma^\wedge)$ and $l^1(\Gamma)^\wedge$ coincide. Indeed, if $x \in (\mathbb{Q}\text{-lin}(A))^\wedge$ is of the form $\sum_{k \in n} r(k) u(k)$ with $n \in \mathbb{N}$, $r : n \rightarrow \mathbb{Q}$, and $u : n \rightarrow A$, then $r^\wedge : n^\wedge \rightarrow \mathbb{Q}^\wedge$, $u^\wedge : n^\wedge \rightarrow A^\wedge$ and $x^\wedge = \sum_{k \in n^\wedge} r^\wedge(k) u^\wedge(k)$; therefore,

$$\|x\|_{l^1(\Gamma)^\wedge} = \|x\|^\wedge = \left(\sum_{k \in n} |r(k)| \right)^\wedge = \sum_{k \in n^\wedge} |r^\wedge(k)| = \|x\|_{l^1(\Gamma^\wedge)}.$$

It follows that \mathcal{L}_1 and $l^1(\Gamma^\wedge)$ are lattice isometric. \triangleright

Corollary 4.4. *Let Q be the Stone representation space of $\mathbb{B} = \mathbb{P}(\Lambda)$. Then the injective Banach lattices $l^1(\Gamma, \Lambda)$ and $C_\#(Q, l^1(\Gamma))$ are lattice \mathbb{B} -isometric.*

\triangleleft This is immediate from Theorem 2.7 and Proposition 4.3. \triangleright

Corollary 4.5. *Given an arbitrary infinite cardinals γ_1 and γ_2 , we may find a Boolean algebra \mathbb{B} such that the injective Banach lattices $l^1(\gamma_1, \Lambda)$ and $l^1(\gamma_2, \Lambda)$ are lattice \mathbb{B} -isometric provided that $\Lambda = \Lambda(\mathbb{B})$. If Q is the Stone representation space of \mathbb{B} then the injective Banach lattices $C_\#(Q, l^1(\gamma_1))$ and $C_\#(Q, l^1(\gamma_2))$ are also lattice \mathbb{B} -isometric.*

\triangleleft The claim follows from Proposition 4.3 and Corollary 4.4 making use of the *cardinal collapsing* phenomena: There exists a complete Boolean algebra \mathbb{B} such that the ordinals γ_1^\wedge and γ_2^\wedge have the same cardinality within $\mathbb{V}^{(\mathbb{B})}$, see [3, 1.13.9]. \triangleright

DEFINITION 4.6. A \mathbb{B} -cyclic Banach lattice X is called *\mathbb{B} -separable*, if there is a sequence $(x_n) \subset X$ such that the norm closed \mathbb{B} -cyclic subspace, generated by the set $\{bx_n : n \in \mathbb{N}, b \in \mathbb{B}\}$, coincides with X . In more detail, X is called \mathbb{B} -separable whenever for every $x \in X$ and $0 < \varepsilon \in \mathbb{R}$ there exist an element $x_\varepsilon \in X$ and a partition of unity $(\pi_n)_{n \in \mathbb{N}}$ in \mathbb{B} such that $\|x - x_\varepsilon\| \leq \varepsilon$ and $\pi_n x = \pi_n x_n$ for all $n \in \mathbb{N}$. It can be easily seen that X is \mathbb{B} -separable if and only if its Boolean valued representation is separable within $\mathbb{V}^{(\mathbb{B})}$. Denote by ω the countable cardinal and put $l^1 := l^1(\omega)$.

Corollary 4.7. *For every infinite cardinal γ , there exists a Stonean space Q such that the injective Banach lattice $C_\#(Q, l^1(\gamma))$ is \mathbb{B} -separable, with \mathbb{B} standing for the Boolean algebra of the characteristic functions of clopen subsets of Q .*

\triangleleft Apply Corollary 4.5 with $\gamma_1 := \gamma$ and $\gamma_2 := \omega$, where ω is the countable cardinal. It follows that $C_\#(Q, l^1(\gamma))$ and $C_\#(Q, l^1(\omega))$ are lattice \mathbb{B} -isometric. Moreover, $\llbracket l^1(\omega^\wedge) \text{ is separable} \rrbracket = 1$ by transfer principle. Taking into account Proposition 4.1 it remains to observe that $\llbracket \mathcal{X} \text{ is separable} \rrbracket = 1$ if and only if $\mathcal{X} \downarrow$ is \mathbb{B} -separable. \triangleright

5. The Main Results

Now we are able to state and prove the main representation and classification results for \mathbb{B} -atomic injective Banach spaces.

DEFINITION 5.1. Let X be an injective Banach lattice. Say that X is *centrally atomic* if X is \mathbb{B} -atomic with $\mathbb{B} = \mathbb{M}(X)$. According to corollary 3.3 this amounts to saying that there is no nonzero element in X disjoint from all Λ -atom, while a Λ -atom is any element $x \in X_+$ such that the principal ideal generated by x is equal to $\Lambda x := \{\lambda x : \lambda \in \Lambda\}$. Given a family of Banach lattices $(X_\gamma, \|\cdot\|_\gamma)_{\gamma \in \Gamma}$, denote by $(\sum_{\gamma \in \Gamma}^\oplus b_\gamma X)_{l^\infty}$ the *l^∞ -sum*, the Banach lattice of all families $\mathbf{x} := (\mathbf{x}(\gamma))_{\gamma \in \Gamma}$ with $\mathbf{x}(\gamma) \in X_\gamma$ for all $\gamma \in \Gamma$ and $\|\mathbf{x}\| := \sup\{\|\mathbf{x}(\gamma)\|_\gamma : \gamma \in \Gamma\} < \infty$.

Lemma 5.2. *For a centrally atomic injective Banach lattice X there exist a set of cardinals Γ and a stable partition of unity $(\pi_\gamma)_{\gamma \in \Gamma}$ in $\mathbb{M}(X)$ such that $\pi_\gamma X$ is purely $(\gamma, \mathbb{B}_\gamma)$ -atomic with $\mathbb{B}_\gamma := [\mathbb{O}, \pi_\gamma]$ for all $\gamma \in \Gamma$ and injective and the representation holds:*

$$X \simeq_{\mathbb{B}} \left(\sum_{\gamma \in \Gamma}^\oplus b_\gamma X \right)_{l^\infty}.$$

◁ This is immediate from Proposition 3.7. ▷

Lemma 5.3. *Suppose that the injective Banach lattices $C_{\#}(Q, l^1(\gamma))$ and $C_{\#}(Q, l^1(\delta))$ are lattice \mathbb{B} -isometric, where Q is the Stone space of \mathbb{B} , while γ and δ are infinite cardinals. If \mathbb{B} is γ -stable and δ -stable then $\gamma = \delta$.*

◁ If $C_{\#}(Q, l^1(\Gamma))$ and $C_{\#}(Q, l^1(\Delta))$ are lattice \mathbb{B} -isometric then $\mathbb{V}^{(\mathbb{B})} \models "l^1(\gamma^{\wedge})$ and $l^1(\delta^{\wedge})$ are lattice isometric" and thus $\mathbb{V}^{(\mathbb{B})} \models \text{card}(\gamma^{\wedge}) = \text{card}(\delta^{\wedge})$. It remains to observe that \mathbb{B} is γ -stable (δ -stable) if and only if $\mathbb{V}^{(\mathbb{B})} \models \text{card}(\gamma^{\wedge}) = \gamma^{\wedge}$ (respectively $\text{card}(\delta^{\wedge}) = \delta^{\wedge}$). ▷

Theorem 5.4. *Let X be a centrally atomic injective Banach lattice. Then there is a set of cardinals Γ and a stable partition of unity $(\pi_{\gamma})_{\gamma \in \Gamma}$ in $\mathbb{B} = \mathbb{M}(X)$ such that the following lattice \mathbb{B} -isometry holds:*

$$X \simeq_{\mathbb{B}} \left(\sum_{\gamma \in \Gamma}^{\oplus} l^1(\gamma, \Lambda_{\gamma}) \right)_{l^{\infty}},$$

where $\Lambda_{\gamma} = \pi_{\gamma} \Lambda$ ($\gamma \in \Gamma$). If a partition of unity $(\rho_{\delta})_{\delta \in \Delta}$ in \mathbb{B} satisfies the same conditions as $(\pi_{\gamma})_{\gamma \in \Gamma}$, then $\Gamma = \Delta$, and $\pi_{\gamma} = \rho_{\gamma}$ for all $\gamma \in \Gamma$.

◁ The required representation follows from Proposition 4.2 and Lemma 5.2.

Assume now that a partition of unity $(\rho_{\delta})_{\delta \in \Delta}$ in \mathbb{B} satisfies the same conditions as $(\pi_{\gamma})_{\gamma \in \Gamma}$. Fix $\delta \in \Delta$ and put $\sigma_{\gamma\delta} := \pi_{\gamma} \rho_{\delta}$ for arbitrary $\gamma \in \Gamma$. If $\sigma_{\gamma\delta} \neq 0$, then the injective Banach lattices $l^1(\gamma, \sigma_{\gamma\delta} \Lambda)$ and $l^1(\delta, \sigma_{\gamma\delta} \Lambda)$ are lattice $[\mathbb{O}, \sigma_{\gamma\delta}]$ -isometric to the same band $\sigma_{\gamma\delta} X$. By Lemma 5.3 $\gamma = \delta$ and thus $\Delta \subset \Gamma$ and $\rho_{\delta} \leq \pi_{\gamma}$ for all $\delta \in \Delta$. Similarly, $\Gamma \subset \Delta$ and $\rho_{\delta} \geq \pi_{\gamma}$ for all $\gamma \in \Gamma$. ▷

Remark 5.5. Let Q be the Stone representation space of \mathbb{B} . Corollary 4.4 enables us to replace $l^1(\gamma, \Lambda_{\gamma})$ by $C_{\#}(Q_{\gamma}, l^1(\gamma))$ in Theorem 5.4 with a stable partition of unity $(Q_{\gamma})_{\gamma \in \Gamma}$ in the Boolean algebra of clopen subsets of Q . Moreover, if some partition of unity $(P_{\delta})_{\delta \in \Delta}$ satisfies the same conditions, then $\Gamma = \Delta$, and $P_{\gamma} = Q_{\gamma}$ for all $\gamma \in \Gamma$.

Corollary 5.6. *Let X be an injective Banach lattice and Q the Stone representation space of $\mathbb{B} = \mathbb{M}(X)$. If X is \mathbb{B} -separable, then X is lattice \mathbb{B} -isometric to $C_{\#}(Q, l^1)$, $l^1 = l^1(\omega)$.*

◁ In Theorem 5.4 each component $l^1(\gamma, \Lambda_{\gamma})$ is \mathbb{B}_{γ} -separable and hence its Boolean valued representation is a separable Banach lattice which is lattice isometric to the internal Banach lattice $l^1(\omega^{\wedge})$. It follows that $l^1(\gamma, \Lambda_{\gamma})$ is lattice \mathbb{B}_{γ} -isometric to $C_{\#}(Q_{\gamma}, l^1)$ for all $\gamma \in \Gamma$ by Proposition 4.1 and Corollary 4.4. From this it is obvious that X is \mathbb{B} -isometric to $C_{\#}(Q, l^1)$. ▷

Proposition 5.7. *A \mathbb{B} -cyclic Banach lattice is atomic if and only if it is \mathbb{B} -atomic and the Boolean algebra \mathbb{B} is atomic.*

◁ The complete Boolean algebra \mathbb{B} is atomic if and only if $\mathbb{B} = \mathcal{P}(A)$ for some set A and then X is the l^{∞} -sum of a family of Banach lattices $(X_a)_{a \in A}$. This l^{∞} -sum is evidently atomic if and only if X_a is atomic for all $a \in A$. ▷

The following corollary should be compared with [7, Theorem 5.6].

Corollary 5.8. *An injective Banach lattice X is atomic if and only if there is a set of cardinals Γ such that the following lattice isometry holds:*

$$X \simeq \left(\sum_{\gamma \in \Gamma}^{\oplus} l^1(\gamma) \right)_{l^{\infty}}.$$

◁ In Remark 5.5 each Q_{γ} is a one-point space by Proposition 5.8 and hence $C_{\#}(Q_{\gamma}, l^1(\gamma))$ is lattice isometric to $l^1(\gamma)$. ▷

Definition 5.9. The partition of unity $(\pi_{\gamma})_{\gamma \in \Gamma}$ in $\mathbb{B} = \mathbb{M}(X)$ satisfying the claim of Theorem 5.4 is called the *decomposition series* of X and is denoted by $d(X)$. Say that

the decomposition series $d(X) = (\pi_\gamma)_{\gamma \in \Gamma}$ and $d(Y) = (\rho_\gamma)_{\gamma \in \Gamma}$ of centrally atomic injective Banach lattices X and Y are *congruent* if there exists a Boolean isomorphism τ from $\mathbb{M}(X)$ onto $\mathbb{M}(Y)$ such that $\tau(\pi_\gamma) = \rho_\gamma$ for all $\gamma \in \Gamma$.

Theorem 5.10. *Centrally atomic injective Banach lattices X and Y are lattice isometric if and only if the Boolean algebras $\mathbb{M}(X)$ and $\mathbb{M}(Y)$ are isomorphic and the decomposition series $d(X)$ and $d(Y)$ are congruent.*

◁ *Sufficiency.* Let X and Y be centrally atomic injective Banach lattices with $d(X) = (\pi_\gamma)_{\gamma \in \Gamma}$ and $d(Y) = (\rho_\gamma)_{\gamma \in \Gamma}$ and let \mathcal{X} and \mathcal{Y} be their respective Boolean valued representations. We identify X and Y with $\mathcal{X} \downarrow$ and $\mathcal{Y} \downarrow$, respectively. Denote $\mathbb{B} := \mathbb{M}(X)$ and $\mathbb{D} := \mathbb{M}(Y)$ and assume that there exists a Boolean isomorphism τ from \mathbb{B} onto \mathbb{D} such that $\tau(\pi_\gamma) = \rho_\gamma$ for all $\gamma \in \Gamma$. Recall that there is a bijective mapping $\tau^* : \mathbb{V}(\mathbb{B}) \rightarrow \mathbb{V}(\mathbb{D})$ such that a ZFC-formula $\varphi(x_1, \dots, x_n)$ is true within $\mathbb{V}(\mathbb{B})$ if and only if $\varphi(\tau^*x_1, \dots, \tau^*x_n)$ is true within $\mathbb{V}(\mathbb{D})$ for all $x_1, \dots, x_n \in \mathbb{V}(\mathbb{B})$, see [3, 1.3.1, 1.3.2, and 1.3.5 (2)]. It follows that $\tau^*(\mathcal{X})$ is an atomic injective Banach lattice within $\mathbb{V}(\mathbb{D})$. Moreover, the mapping $x \mapsto \tau^*(x)$ ($x \in \mathcal{X} \downarrow$) is a lattice isometry from $\mathcal{X} \downarrow$ onto $\tau^*(\mathcal{X}) \downarrow$. If $\alpha = \text{card}(\text{at}_1(\mathcal{X}))$ and $\beta = \text{card}(\text{at}_1(\mathcal{Y}))$, then $\tau^*(\alpha) = \text{mix}_{\gamma \in \Gamma} \tau(\pi_\gamma) \gamma^\wedge$ and $\beta = \text{mix}_{\gamma \in \Gamma} \rho_\gamma \gamma^\wedge$, so that $\beta = \tau^*(\alpha)$. By [3, 1.3.5 (2)] we have $\tau^*(\alpha) = \text{card}(\text{at}_1(\tau^*(\mathcal{X})))$ and $\text{card}(\text{at}_1(\mathcal{Y})) = \text{card}(\text{at}_1(\tau^*(\mathcal{X})))$. It follows that $\tau^*(\mathcal{X})$ and \mathcal{Y} are lattice isometric and hence $\tau^*(\mathcal{X}) \downarrow$ and $\mathcal{Y} \downarrow$ are lattice \mathbb{B} -isometric.

Necessity. Suppose that h is a lattice isomorphism from X onto Y . Then the mapping τ from \mathbb{B} onto \mathbb{D} defined by $\tau(\pi) = h \circ \pi \circ h^{-1}$ is a Boolean isomorphism. Moreover, $h(\mathbb{B}\text{-at}_1(\pi X)) = \mathbb{B}\text{-at}_1(\tau(\pi)Y)$. Now it can be easily verified that πX is $([\mathbb{O}, \pi], \gamma)$ -atomic if and only if $\tau(\pi)Y$ is $([\mathbb{O}, \tau(\pi)], \gamma)$ -atomic. It follows that $d(X)$ and $d(Y)$ are congruent. ▷

Corollary 5.11. *Let X be a centrally atomic injective Banach lattice. Then there is a family of Stonean spaces $(Q_\gamma)_{\gamma \in \Gamma}$, with Γ a set of cardinals, such that Q_γ is γ -stable for all $\gamma \in \Gamma$ and the following lattice \mathbb{B} -isometry holds:*

$$X \simeq_{\mathbb{B}} \left(\sum_{\gamma \in \Gamma}^{\oplus} C_{\#}(Q_\gamma, l^1(\gamma)) \right)_{l^\infty}.$$

If some family $(P_\delta)_{\delta \in \Delta}$ of Stonean spaces satisfies the above conditions, then $\Gamma = \Delta$, and P_γ is homeomorphic with Q_γ for all $\gamma \in \Gamma$.

◁ This is immediate from Theorem 5.10 and since Corollary 4.4 (see Remark 5.5). ▷

DEFINITION 5.12. The *second \mathbb{B} -dual* of a \mathbb{B} -cyclic Banach space is defined by $X^{\#\#} := (X^\#)^\# := \mathcal{L}_{\mathbb{B}}(X^\#, \Lambda)$. A \mathbb{B} -cyclic Banach space is said to be *\mathbb{B} -reflexive* if the image of X under the *canonical embedding* $X \rightarrow X^{\#\#}$ coincide with $X^{\#\#}$, see [3, p. 316].

Theorem 5.13. *Let X be a \mathbb{B} -reflexive injective Banach lattice with $\mathbb{B} = \mathbb{M}(X)$. Then there are a sequence of Stonean spaces $(Q_k)_{k \in \mathbb{N}}$, and an increasing sequence of naturals (n_k) such that the following lattice \mathbb{B} -isometry holds:*

$$X \simeq \left(\sum_{k \in \mathbb{N}}^{\oplus} C_{\#}(Q_k, l^1(n_k)) \right)_{l^\infty}.$$

If some family $(P_k)_{k \in \mathbb{N}}$ of Stonean spaces satisfies the above conditions, then Q_k and P_k are homeomorphic for all $k \in \mathbb{N}$.

◁ Again identify X with $\mathcal{X} \downarrow$, where \mathcal{X} is an *AL-space* in $\mathbb{V}(\mathbb{B})$. It follows from Theorem [3, Theorem 5.8.12] that $\mathcal{X}^* \downarrow = \mathcal{X} \downarrow^\#$ and $\mathcal{X}^{**} \downarrow = \mathcal{X} \downarrow^{\#\#}$. Therefore, X is \mathbb{B} -reflexive if and only if $\llbracket \mathcal{X} \text{ is reflexive} \rrbracket = 1$. Since a reflexive *AL-space* is finite-dimensional, we have

$$1 = \llbracket (\exists n \in \mathbb{N}^\wedge) \dim(\mathcal{X}) = n \rrbracket = \bigvee_{n \in \mathbb{N}} \llbracket \dim(\mathcal{X}) = n^\wedge \rrbracket.$$

This relation enables us to choose a countable partition of unity (b_n) in \mathbb{B} such that $b_n \leq [\mathcal{X}$ is a n^\wedge -dimensional AL -space]. Pick the sequence (n_k) of indices of nonzero projections in (b_n) and denote by Q_k the Stonean space of a Boolean algebra $\mathbb{B}_k := [\mathbb{O}, b_{n_k}]$. Now, by the Transfer Principle we conclude that $\mathbb{V}^{(\mathbb{B}_k)} \models "b_{n_k} \wedge \mathcal{X} \text{ is lattice isometric to } l^1(n_k^\wedge)"$. The proof is concluded with the help of Theorem 5.10 taking into consideration that for each finite cardinal γ every complete Boolean algebra is γ -stable and γ^\wedge is a finite cardinal within $\mathbb{V}^{(\mathbb{B})}$. \triangleright

References

1. Kusraev A. G. Boolean valued analysis and injective Banach lattices // Dokl. RAS.—2012.—Vol. 444, № 2.—P. 143–145.
2. Kusraev A. G. Classification of injective Banach lattices // Dokl. RAS.—2013.—Vol. 453, № 1.—P. 12–16.
3. Kusraev A. G. and Kutateladze S. S. Boolean Valued Analysis: Selected Topics.—Vladikavkaz: Vladikavkaz Scientific Center Press, 2014.—iv+400 p.—(Trends in Science. The South of Russia).
4. Aliprantis C. D. and Burkinshaw O. Positive Operators.—London etc.: Acad. Press Inc., 1985.—xvi+367 p.
5. Meyer-Nieberg P. Banach Lattices.—Berlin etc.: Springer, 1991.—xvi+395 p.
6. Lotz H. P. Extensions and liftings of positive linear mappings on Banach lattices // Trans. Amer. Math. Soc.—1975.—Vol. 211.—P. 85–100.
7. Cartwright D. I. Extension of positive operators between Banach lattices // Memoirs Amer. Math. Soc.—1975.—164 p.
8. Haydon R. Injective Banach lattices // Math. Z.—1974.—Vol. 156.—P. 19–47.
9. Kusraev A. G. A Boolean transfer principle for injective Banach lattices // Sib. Mat. Zh.—2015.—Vol. 56, № 5.—[DOI 10.17377/smzh.2015.56.511].
10. Kusraev A. G. and Kutateladze S. S. Introduction to Boolean Valued Analysis.—M.: Nauka, 2005.—526 p.—[in Russian].
11. Gordon E. I. Real numbers in Boolean-valued models of set theory and K -spaces // Dokl. Akad. Nauk SSSR.—1977.—Vol. 237, № 4.—P. 773–775.
12. Kusraev A. G. Dominated Operators.—M.: Nauka, 2003.—619 p.—[in Russian].
13. Luxemburg W. A. J. and Zaanen A. C. Riesz Spaces. Vol. 1.—Amsterdam and London: North-Holland, 1971.—514 p.

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KUSRAEV ANATOLY GEORGIEVICH
Vladikavkaz Science Center of the RAS, *Chairman*
22 Markus Street, Vladikavkaz, 362027, Russia;
K. L. Khetagurov North Ossetian State University
44–46 Vatutin Street, Vladikavkaz, 362025, Russia
E-mail: kusraev@smath.ru

АТОМИЧНОСТЬ В ИНЪЕКТИВНЫХ БАНАХОВЫХ РЕШЕТКАХ

Кусраев А. Г.

Цель заметки — рассмотреть булевозначную интерпретацию понятия атомической банаховой решетки и дать полное описание соответствующего класса инъективных банаховых решеток.

Ключевые слова: инъективная банахова решетка, атомическая банахова решетка, булевозначное представление, классификация.