Section 6. Technical sciences

<https://doi.org/10.29013/ESR-19-9.10-35-38>

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ALGORITHM FOR USING THE RULE OF THREE SIGMS IN PROCESSING RESULTS OF MEASUREMENTS AND INCREASING THE RELIABILITY OF EXPERIMENTAL DATA

Abstract. Practical computations per unit of measurement deviation of a random variable from the true value are considered. The basic formulas are shown for checking on normal law-and-extremal data and the corresponding reliability is given $P(\sigma) = 0.6826$, $P(2\sigma) = 0.9524$, $P(3\sigma) = 0.9972$. As well as specific examples, a detailed order and algorithm for using the three sigma rule is shown.

Keywords: three sigma rule, processing of measurement results, standard deviation, scattering, center scattering, random variable, mathematical expectation, normal distribution law, reliability of measurements, true value, real value.

In practical calculations, the standard deviation σ is taken as the unit of measurement of the deviation of a random variable subject to the normal law

from its scattering center. Then, on the basis of the formula, we obtain the equalities useful in various calculations

$$P(-\sigma < \bar{x} < \sigma) = \Phi\left(\frac{1}{\sqrt{2}}\right) = 0.6826,$$

$$P(-3\sigma < \bar{x} < 3\sigma) = \Phi\left(\frac{3}{\sqrt{2}}\right) = 0.9972.$$

$$P(-2\sigma < \bar{x} < 2\sigma) = \Phi(\sqrt{2}) = 0.9544,$$

These results are geometrically depicted in (Figure 1).

Figure 1. Scattering of measurement results under normal distribution law

The figure shows that within one standard deviation lies 68.26% of the values taken by a normally distributed random variable (corresponds to the fraction of the area under the distribution curve). Within two standard deviations – already 95.44%, and within three – 99.72%. This means that the probability that a random variable takes a value deviating from the mathematical expectation by more than three standard deviations does not exceed 0.28%, i.e. negligible.

The “three sigma” criterion is applied to measurement results distributed according to the normal law.

This criterion is reliable with the number of measurements $n > 20 \dots 50$. The arithmetic mean $\bar{X}$ and the standard deviation σ are calculated without taking into account the extreme (suspicious) values of X_i . In this case, the gross error (miss) is the result of X_i , if the difference $|\bar{X} - X_i|$ – exceeds 3σ .

Table 1 shows the experimental work in measuring the moisture content of materials. According to a priori information, the measurement result obeys the normal law of probability distribution. Checks the source data before processing the measurement results.

Table 1. – The results of experimental work in measuring the moisture content of materials

№	$W_{i,\%}$	$W_{\text{ср.в.},\%}$	$\Delta = W_i - W_{\text{ср.в.}}$	$\Delta^2, 10^{-4}$
1	2	3	4	5
1.	17.80	17.88	0.08	64
2.	17.65		0.18	324
3.	17.70		0.18	324
4.	17.88		0	0

1	2	3	4	5
5.	17.90	17.88	0.02	4
6.	18.15		0.27	729
7.	18.25		0.37	1369
8.	17.95		0.07	49
9.	17.65		0.23	529
10.	17.75		0.13	169
11.	18.70		0.82	6724
12.	17.70		0.18	324
13.	17.98		0.10	100
14.	18.28		0.40	1600
15.	18.25		0.37	1369
16.	18.20		0.32	1024
17.	17.60		0.28	784
18.	17.50		0.38	1444
19.	17.35		0.53	2809
20.	17.40		0.48	2304

Checking the normal law of distribution of moisture measurement results.

Estimated standard deviation (σ_{15}).

$$\sigma_{15} = \sqrt{\frac{\left(64 + 324 + 324 + 4 + 729 + 1369 + 49 + 529 + 169 + 6724 + 324 + 100 + 1600 + 1369 + 1024 + 784 + 1444 + 2809 + 2304\right) \cdot 10^{-4}}{15 \cdot 14}} = 0.10$$

We calculate $3\sigma_{15} = 0.30$. More than $3\sigma_{15}$ from the arithmetic mean differs 7, 11, 14, 15, 19, 20 th value. Therefore, it is erroneous and should be discarded. Validation continues until three sigma runs.

We define the numerical characteristics of the normal distribution with density

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

Mathematical expectation and variance

$$M(X) = \int_{-\infty}^{+\infty} xf(x)dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} xe^{-\frac{(x-m)^2}{2\sigma^2}} dx = \begin{cases} \frac{x-m}{\sigma\sqrt{2}} = t & dx = \sigma\sqrt{2}dt \\ \frac{1}{\sigma\sqrt{2}} dx = dt & x = m + \sigma\sqrt{2}t \end{cases}$$

$$\left. \frac{x}{t} \right|_{-\infty}^{+\infty} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} (m + \sigma\sqrt{2}t) e^{-t^2} \sigma\sqrt{2}dt = \frac{\sigma\sqrt{2}}{\sqrt{\pi}} \int_{-\infty}^{+\infty} te^{-t^2} dt + \frac{m}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-t^2} dt$$

As

$$\int_{-\infty}^{+\infty} te^{-t^2} dt = -\frac{1}{2} \int_{-\infty}^{+\infty} e^{-t^2} d(-t^2) = -\frac{1}{2} e^{-t^2} \Big|_{-\infty}^{+\infty} = 0$$

$$\int_{-\infty}^{+\infty} e^{-t^2} dt = 2 \int_0^{+\infty} e^{-t^2} dt = \sqrt{\pi}$$

is the Poisson integral, then we finally obtain

$$M(X) = \frac{m}{\sqrt{\pi}} \sqrt{\pi} = m.$$

The quantity m is a normally distributed random variable X , called its “center of dispersion.”

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